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DETERMINATION OF "STRUCTURE-OF-INTELLECT" ABILITIES INVOLVED
IN NINTH-GRADE ALGEBRA AND GENERAL MATHEMATICS.

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NEW INTELLECTUAL APTITUDES WERE INVESTIGATED TO
DETERMINE TO WHAT EXTENT THEY INFLUENCE ACHIEVEMENT IN
NINTH-GRADE ALGEBRA AND GENERAL MATHEMATICS COURSES. TESTS
AND CRITERIA WERE COMPLETED FOR APPROXIMATELY 400 STUDENTS IN
THE NINTH GRADE. THEY WERE ENROLLED IN ONE OF FOUR
MATHEMATICS COURSES AND ADMINISTERED ACHIEVEMENT EXAMINATIONS
UPON COMPLETION. THE ROLES OF SEPARATE FACTORS WERE INFERRED
FROM THE BETA WEIGHTS OF THE EQUATIONS. SOME OF THE
CONCLUSIONS WERE THAT STANDARD TESTS DO AS WELL AS FACTOR
TESTS IN PREDICTION, AND THAT COURSE LEVEL INFLUENCES THE
FACTORS INVOLVED. FURTHER RESEARCH IS SUGGESTED TO STUDY
APTITUDE WITH A BROADER SAMPLING OF INTELLECTUAL ABILITIES.
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NINTH-GRADE ALGEBRA AND GENERAL MATHEMATICS

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Studies of Aptitudes of High-level Personnel

REPORTS FROM THE PSYCHOLOGICAL LABORATORY
THE UNIVERSITY OF SOUTHERN CALIFORNIA

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INTRODUCTION

In these days of rapid technological changes, the accelerated rate with which problems of all kinds arise has placed great demands upon human thinkers. The appetite for better prepared and more capable scientists and engineers continues unappeased. Education's role has been to find and to develop students with potential for making contributions in those fields. Basic to the training of a scientist or technologist is training in mathematics; a key subject in all those areas. The discovery of which young people have potential for mastering mathematical subjects, then, is an important step in channeling those with appropriate talents in the direction of science and the technologies.

For the student who faces the early point of decision as to whether or not a career in some science or technology is suitable, and for his educational adviser, one very critical time is at the beginning of the ninth grade, when a choice is ordinarily made to take college-preparatory algebra or not to take it. In making this decision, the student and his adviser are usually lacking a great deal of information regarding the student's potential for success in beginning algebra and in mathematics in general. Algebra is so basic to more advanced study of mathematics that success or failure at that point in the student's career is rather decisive regarding future potential for mathematics and for the fields that depend upon it.

Anything that can be done, therefore, toward a better guidance of students into ninth-grade algebra, or away from the subject, would be of great value to the student and to his adviser. Letting students just try algebra and either "sink or swim" is not the optimal solution from the standpoint of the best use of time or from the standpoint of mental health. Although ordinary academic-aptitude tests are somewhat prognostic of success in beginning algebra, they are definitely less successful in that direction than they are in connection with courses of a more verbal character. Even the less verbal parts of ordinary academic-aptitude tests fall short of good prediction, probably because they do not assess the more pertinent abilities upon which success in algebra depends.

Recent explorations of intellectual aptitudes on a vast scale (Guilford and Merrifield, 1960) have revealed numerous kinds of intellectual abilities that were not even suspected before and certainly were not taken into account when academic-aptitude tests were devised. It should be a well-accepted principle by now that if we are to improve prediction of success in some particular human endeavor by means of tests, we cannot expect gains from giving simply more tests of the same kinds that we already have. The only hope of achieving substantial improvement is from bringing into the picture tests of other aptitude factors previously neglected. That is precisely what this study is about. The goal is to consider new intellectual aptitudes that should logically be promising in the prediction of success in ninth-grade algebra and to see whether assessments of those abilities would add to predictions made from standard academic-aptitude tests.

The strategy of the study is actually broader than this. It reflects a new philosophy of test validation that emphasizes a thoroughly rational basis. One of the authors (Guilford, 1948) has repeatedly emphasized the importance of conducting validation studies with as much knowledge as possible of the nature of the psychological variables involved, in other words, of considering construct validity in connection with studies of predictive validity. He has further maintained that the best possible information regarding construct validity is in terms of the factor-analytic information regarding tests, where that information is appropriately derived.

In a predictive-validation study, not only is it well to keep informed, but there are also statistical advantages in the factorial approach. In a multiple-prediction approach, with the use of multiple-regression equations, the best way of handling the problem of linear restraints is to use factor scores. Factor scores, although not always uncorrelated, are minimally intercorrelated, a condition favorable for high multiple correlation, everything else being equal.

There have been a few pertinent investigations that have given attention to a factorial approach (Weber, 1953; Kline, 1956; Werdelin, 1958; and Majewska, 1960). As we shall see, although these studies went various distances along the route that we have followed and we have depended upon them in planning this investigation, they were lacking two important things—full information regarding the possible range of intellectual abilities that might be brought into consideration, and a comprehensive theory such as the structure-of-intellect model provides, to serve as a frame of reference. We have been able to correct those shortcomings to some extent, although, as we shall see, we were lacking information about, and tests for, some of the hypothesized structure-of-intellect factors that may be pertinent.

HYPOTHESES INVESTIGATED

The Basic, General Hypothesis

The major emphasis in this investigation was to attempt to obtain a picture of aptitude for ninth-grade algebra in terms of known intellectual factors. Since the high school in which the subjects were found taught two courses under the heading of General Mathematics to about half of its ninth-grade students, there was a secondary interest in deriving a similar understanding of aptitude for success in those courses, also. There is interest in some comparisons of the two aptitude patterns, as a basis for future classification of students in the one course versus the other. Although the major interest was the more basic one of determining the factors related to the two kinds of criteria, algebra and general mathematics, it was hoped and expected that much better batteries for differentiating the two kinds of aptitude would be forthcoming as a natural outcome of this study.

There were some precedents for expecting that meaningful descriptions of criteria would be forthcoming from this study. One outcome of the AAF Aviation Psychology Research Program during World War II was an analysis of variances of criteria of success in pilot training and navigator training into component proportions of variance attributable to a number of aptitude factors (Guilford, 1948). The ability to solve arithmetical problems in an arithmetic-reasoning test has also been described (Guilford, 1960) in terms of contributions of the factors of general reasoning (cognition of semantic systems, in terms of structure-of-intellect concepts), numerical facility (memory for symbolic implications), verbal comprehension (cognition of semantic units), visualization (cognition of figural transformations), and spatial orientation (cognition of figural systems). Ability to perform a task of formulating plans (Berger, et al, 1957) has been analyzable into portions attributable to factors of semantic elaboration (divergent production of semantic implications), originality (divergent production of semantic transformations), judgment (possibly evaluation of semantic transformations), and ideational fluency (divergent production of semantic units). Ability to master some simple, semantic problem-solving tasks were analyzed (Merrifield, et al, 1962) into components recognized as verbal comprehension (cognition of semantic units), conceptual foresight (cognition of semantic implications), originality (divergent production of semantic transformations), and sensitivity to problems (evaluation of semantic implications). Thus, there is considerable precedent for the kinds of analyses that were attempted in this investigation.

Some Specific Hypotheses

We now focus our attention upon aptitude for algebra, and in doing so we shall draw upon the findings of previous investigators. The results from the previous studies do not so much give us a factorial picture of achievement in algebra as they provide us with a factorial picture of the tests used in validation studies. The tests were factor analyzed, but not all of them were necessarily valid for predicting achievement in algebra. We are thus able to see in terms of factor content what kinds of tests other investigators have elected to try out as potentially valid for predicting mathematical achievement. This knowledge helps to guide the search for predictive tests, but does not take into account all logically potentially predictive tests and relevant factors.

Structure-of-Intellect Categories. — At this point, in order to make more meaningful the discussion to follow, we digress sufficiently to remind the reader concerning the major concepts connected with the structure of intellect. Much fuller descriptions are to be found elsewhere (Guilford, 1959; Guilford and Merrifield, 1960). In the system, each intellectual factor is defined in terms of three items of information, concerning three dimensions of the model—operation, content, and product. The kinds of operations and their letter symbols are given in the first column of the following table, the kinds of contents in the second column, and the kinds of products of information in the third:

Operations		Contents		Products	
Cognition -----	C	Figural -----	F	Units -----	U
Memory -----	M	Symbolic -----	S	Classes -----	C
Divergent production ---	D	Semantic -----	M	Relations -----	R
Convergent production --	N	Behavioral --	B	Systems -----	S
Evaluation -----	E			Transformations -----	T
				Implications -----	I

When the kind of operation, kind of content, and kind of product are specified for a factor, a unique ability is defined. Thus, the trigram MSI stands for memory for symbolic implications, more commonly known as the numerical-facility factor. This factor pertains to implications, because in a numerical operation like $2 \times 5 = 10$, the given, 2×5 , implies 10. "If 2×5 , then 10," is the implied statement. It is a memory ability because the implications have been previously practiced and are recalled in precisely the same way in which they were memorized.

Factors Previously Investigated. — The four authors, previously mentioned, did not attempt to interpret their factors in terms of structure-of-intellect concepts, because those concepts were not generally public information until their studies had been completed. We have attempted to supply this interpretation, in order to put their findings within our frame of reference. This operation was logically possible where they obtained factors recognized as traditional and already located within the structure-of-intellect model.¹ Another link is in terms of the nature of their described tests for each factor and the SI factors that have been found related to such tests. Examples of such instances will be given.

The earliest of these studies (Weber, 1953) aimed to determine the factor composition of various kinds of tests involving numbers; no criterion of mathematics achievement being involved. From 14 tests administered to 100 boys and 85 girls at the 15-year-old age level, six factors were extracted and rotated, five of which were interpreted in line with Meili's factors (Meili, 1946): numerical facility, complexity, fluency, integration, and flexibility. Of these, the numerical-facility factor appears to be in line with the traditional factor N, defined as factor MSI in the structure of intellect. The other factors, from the somewhat heterogeneous lists of tests defining them, appear to be confoundings of structure-of-intellect factors. "Complexity" may be a confounding of CFR and NFR (suggested by a Geometric Analogies test), with CSR and NSR (suggested by a Number Analogies test), with CSS (suggested by a Number Series test), and NMS (suggested by a Picture Ordering test). No wonder the factor was called "complexity"!

Weber's "fluency" factor appears to be primarily a confounding of two divergent-production factors. His test Designs, which involves making different organizations of the same lines, appears to be a measure of factor

¹ Hereafter, for the sake of brevity, the structure of intellect will often be indicated by the symbol SI.

DFS, and his test Sentences, which involves giving a number of sentences in response to the same initial letters for words, appears to be a measure of factor DMS, known as expressional fluency.

The factor called "integration" is defined in a way that suggests divergent production of transformations. But the leading tests are likely to measure other kinds of factors, including CMS (Practical Problems is an arithmetical-reasoning test, a most constant indicator of CMS). The factor "flexibility" had a strong relationship with an apparent CSC test (e.g., one test contains items involving the task: from a set of numbers, pick out multiples of 3).

The factor called a "residual" had two loadings as high as .50 (one other factor not called a residual had no loadings that high) and seems interpretable as a confounding of factors NMI and EMR (probably measured by the test Logical Problems, a syllogism test) with NMS (for the test Picture Order). It is quite clear, from the writers' point of view, that Weber had far too many factors for the number of tests in the battery, hence the extensive merging of basic factors, either because they are actually correlated in his population or because his tests straddled two or more factors in common.

Of the four studies considered here, that by Kline was the best designed, particularly in handling the mathematics-criterion problem (Kline, 1956). Among the criterion variables were 18 especially designed algebra tests, each emphasizing a certain type of problem. There were also a final-examination score and a semester grade. There were 17 marker tests for expected factors, but, as in the case of Weber's study, there were too many factors involved for the number of tests. Kline extracted 12 factors, but considered only 5 of them usable because 7 failed to show sufficient congruency in results from two different samples of 129 students each (boys in intermediate algebra).

Of the interpreted factors, one was identified properly as CMU. Although CMU variance probably dominated the factor, there are probably other factors confounded to some degree, for in the list were tests like Similes and Inventive Opposites (both of the type that often measure factor DMR), and Opposites, that could measure CMR. Five of the algebra tests were significantly loaded on this verbal factor, including one emphasizing knowledge of algebraic concepts, two emphasizing translating verbal statements into symbolic statements, and two being composed of word problems to be solved by using algebraic equations.

The factor identified as "deductive reasoning" is probably a composite of factors CFR, CFS, and CFT, for in the list were the tests Figure Analogies (CFR), Block Counting (CFS and CFT), and Similar Figures (like Thurstone's Lozenges test, in principle). The final-examination and semester-grade criteria were both significantly related to this factor.

A factor identified as "algebraic manipulative skill" was confined to six algebra tests. The most heavily loaded types of algebra tests pertained to simplifying and factoring operations, which suggests that the factor CST (hypothesized by the SI theory but not yet demonstrated empirically) may be heavily involved. The student must see what changes or reorganizations must be made in an algebraic expression in order to simplify or to factor it.

The very familiar numerical-facility factor was well determined and fairly clear. Curiously, no algebra tests or grades were significantly related to this factor.

The title of the factor called "adaptability to a new task" has very little suggestive value concerning the psychological properties involved. It was so named because of the heterogeneous appearance of the list of tests weighted on it. Every test loaded most strongly on the factor can be identified with a different SI factor; Similes (DMR), Block Counting (CFS and CFT), Family Trees (NMI), Similar Figures (CFT), and Word-Word, which called for memorizing word pairs, (MMI, a factor as yet not demonstrated).

On the whole, one gains the impression that in Kline's study very few of the aptitude factors were significantly related to the algebra criteria, and of all the basic intellectual factors, only the "deductive-reasoning" factor and CMU had many such relationships. It is surprising to find so much of a hiatus between scores in algebraic-operations tests and final-examination scores and semester grades, both in regard to their lack of relationships to one another and in regard to their affiliations with different factors. This outcome definitely raises the important question as to what the algebra criterion should be.

The Werdelin study was conducted in Swedish high schools (Werdelin, 1958). The subjects were 217 15-year-old boys. No statement was made as to the kinds of mathematics courses involved. Some mathematics tests given were treated as predictors, with grades as the criterion of achievement. About 30 intellectual-aptitude tests were analyzed, from which only six factors were extracted and rotated, one of which was regarded as a residual, in spite of one loading of .50. More than six factors were undoubtedly represented among 30 tests, and the factors that were found are probably composites, with one traditional factor salient in each of most of them, for example, the factors CMU, MSI, CFS (confounded with CFT), and EMR. One factor was called "general mathematical" because its heaviest loadings were in Plane Geometry, Solid Geometry, and three Arithmetic tests. The dominant factor in this composite might be CMS.

In line with the strategy in the present study, Werdelin proceeded to derive factor scores to correlate with the grade criteria. Each factor composite was made up of only the few best tests of each factor, weighted in a multiple-regression equation. Factor R (the mathematical composite) naturally had the highest correlations, ranging from .3 to .8 in different courses. Some readers may well regard this kind of correlation as being between two kinds of criteria, rather than between an aptitude score and a criterion score. The composite for

factor D (deductive) correlated .3 to .6; that for factor S (space and visualization) correlated .1 to .7; that for the verbal factor, -.1 to .4; and that for the numerical factor, .1 to .6. In the last respect, Werdelin's results are contradictory to those of Kline.

Majewska's objective (1960) was to determine the factor composition of a wide variety of tests involving number work of different kinds. Her subjects were 150 girls in the range 15 to 17 years of age. All had completed algebra and most had completed plane geometry. The battery of tests analyzed contained 10 standard instruments, such as parts of the Thurstone PMA battery, the Holzinger-Crowder Unifactor tests, and two parts of the California Mathematics Tests; 12 adapted tests selected to help determine the factor structure; and 14 new tests, emphasizing numerical and other symbolic content. From the use of this battery 12 factors were extracted and rotated by the oblimax method, followed by graphic revisions.

Majewska did not name her factors, but gave interpretations. From the leading tests on each factor it is possible to suggest which SI factor or factors (for some are probably confoundings) are dominant, without expecting that Majewska would agree at all points. Her number factor (A) is well determined and probably fairly unique, being defined by the usual tests involving numerical operations. Factor CSS (her factor B) seems to be revealed by the tests Number Oddities (involving seeing a principle in a set of equations) and PMA Reasoning, involving letter series.

Factor CFC is suggested by her factor C, since the leading test is Figure Grouping, involving saying which of several forms does not belong to a set. There may be some confounding with other factors, for Figure Grouping stands very much alone in suggesting this factor.

Her factor D appears to be a confounding especially of NSR, NSI, and EMR, for its leading tests involve drawing conclusions from symbolic statements in three cases and verbal syllogisms in another. Factor CMS (her factor F) seems clearly brought out by several tests of arithmetical reasoning, in various forms. CMU (her factor H) is clearly tagged by the PMA Verbal Meaning test. NSR (factor G) is suggested by Algebraic Inequalities, involving deducing an inequality from two simple algebraic statements, and by Formulas and Figures, involving associating algebraic statements with figures. CFR is suggested by a figure-analogies test called Figure Changes, but her factor J is probably more complex than that. Factor DSU (her factor L) is well determined by PMA Word Fluency and by a new test analogous to it called "Number Fluency," which requires the examinee to give several numbers to satisfy a specified category. Incidentally, this is the only instance, to our knowledge, in which Thurstone's factor of "word fluency" has been shown to be measured by a test composed of numbers. This finding supports the more general conception of the factor as the divergent production of symbolic units, where the units may be composed of either letters or numbers.

The two "mathematical" tests had significant loadings in only one factor, the one recognized as CMS or more commonly known as "general reasoning." Perhaps this reflects an emphasis upon arithmetic in those two tests, but in view of this, it is surprising that the relationships to the number factor were essentially zero. This may be because liberal time is allowed in the two tests so that numerical errors can be corrected.

A net impression from all of these studies is that the relations between factor measures and achievement in mathematics have been rather limited; that factors CMU and CMS, the old standbys of general aptitude tests, have fared better in this respect than have other factors. But it cannot be said that all other promising factors have been well represented or that criteria have emphasized algebra, except in the study by Kline. There is still room for investigation along factorial lines, where factors are more broadly and systematically sampled, and where algebra-achievement criteria are well chosen.

Relative Attention Given to Various Factors. —For a rapid survey of attention given to various factors by earlier investigators, Table I will be of use. The 27 factors believed to be represented to appreciable extents in their tests are indicated by columns of the matrix, and wherever a factor was believed to make a significant contribution to a test for an author's obtained factor, an x is recorded. The frequency with which each factor was believed to be represented is indicated in the bottom row of the table.

Of the 27 factors, 11 were in the cognition category, with the memory and evaluation categories being barely mentioned. As to content, the figural, symbolic, and semantic categories are about equally represented; the behavioral (social intellectual) category was suspected in one test. Of the six product categories, relations and systems were about equally emphasized, with each of the others usually receiving about half as much attention. As to single factors, CFR (cognition of figural relations) and MSI (numerical facility) appeared seven times each. CSS (cognition of symbolic systems), CMS (cognition of semantic systems), NSR (convergent production of symbolic correlates), and EMR (evaluation of semantic relations) appeared five times each.

In evaluation of the distributions of attention to various types of abilities, a number of points can be suggested. As for different kinds of operations, it is probably correct to place much weight upon cognition, for comprehension of problems has been shown to be important in previous factor-analytical studies of problem solving. Much weight should also be given to convergent-production abilities, since answers to mathematical problems are fully determined, if enough information is given. Memory abilities are of an unknown status with respect to school learning. In general, much more attention should be paid to this problem. Because of this general uncertainty, we included only one memory ability, the well-known factor of numerical facility, which is now classified as the factor for memory for symbolic implications. This interpretation is well supported by the fact that both numerical facility and the factor that is strongest in the Wechsler Digit-Symbol test are one and the same (Davis, 1956; deMille, 1962).

Table 1

Cross Index of Structure-of-Intellect Factors
and
Factors Obtained from Reference Studies

Reference factors	Structure-of-Intellect Factors																			
	CFC	CFR	CFSS	CFST	CSC	CSSR	CSSS	CSSS	CSSU	CSSU	CSSU	CSSU	CSSU	CSSU	CSSU	CSSU	CSSU	CSSU	CSSU	CSSU
WEBER																				
Numerical Facility																				
Complexity																				
Fluency																				
Integration																				
Flexibility																				
KLINE																				
Verbal Comprehension																				
Deductive Reasoning																				
Algebraic Manipulation*																				
Number Ability																				
Adaptability																				
WERDELIN																				
Numerical																				
Verbal																				
Visual																				
Deductive																				
Reasoning																				
MAJEWSKA																				
A																				
B																				
C																				
D																				
E																				
F																				
G																				
H																				
I																				
J																				
K																				
L																				
Residual																				
Total Times Each Factor Found	1	7	4	4	1	4	5	2	1	5	1	7	1	1	2	1	1	3	1	5

* Only algebra tests were loaded on this factor.

In the area of content or general kind of information, it would appear that both the semantic and the figural abilities were over-represented in previous studies. Two semantic abilities, CMU and CMS, ordinarily the dominant ones in traditional academic-aptitude scales, should, of course, be included in almost any academic-aptitude study. In the study reported here, they would almost certainly be pertinent in predicting achievement in general mathematics. They might well make some contribution to achievement in algebra, since instruction in algebra is in terms of verbal communication. Figural abilities probably dominate the scene for aptitude for the subject of geometry. But, as stated before, when algebraic aptitude is considered, the prevailing kind of test and factor should be in the symbolic area.

As for the product categories, tests involving units of information and classes of information would seem to be most dispensable in connection with traditional algebra. Exceptions to this idea might be that the most capable mathematical thinker can more readily form symbolic units, in grasping whole expressions and even whole equations as units. This hypothesis should receive attention at some time, but there would seem to be more promising abilities, at least at the elementary levels of mathematics. The use of classes might be important in terms of recognition of types of expressions and types of equations, but probably this need is more luxurious and occasional than fundamental and persistent. The need for emphasis on relations and systems is definitely reasonable; the need for much more attention to transformations and implications would seem to be essential in order to get to the heart of algebraic manipulations.

Factors to be Included in this Investigation

As in many studies of this kind, there had to be some limitation on the number of factors that could be included, owing to limits in testing time with the subjects. In some ten hours of aptitude testing, 19 factors were included, out of a total of more than 60 that possibly could have been represented in terms of available tests. Severe selection was therefore necessary. Adding to testing time was the consideration that tests for the previously demonstrated factors (or for those under investigation by the Aptitudes Project, at the time, in parallel studies) should be re-analyzed in the samples of this study. With a very few exceptions, the tests had not been previously analyzed in a ninth-grade population. Even if they had been, it would be desirable to re-analyze them in this particular study, in order to determine whether the tests were all functioning factor-wise as expected, and thus vindicate the choices of tests to represent each factor in validation.

Analyses were made in the algebra and general-mathematics samples separately, also in order to determine how factor scores should be derived in the two groups. Since, for the multiple-regression analysis, boys and girls were to be kept together in order to have samples of acceptable size, factor analyses were carried out for samples of boys and girls separately, at least to the point at which it was possible to determine whether similar factor structures prevailed for the two sexes. In one previous study (Guilford, et al, 1961) it was found that for most factors in the divergent-production area, primarily, the two sexes had very similar factor structures, but with respect to one factor there was a difference. Sex differences in structure in this investigation, if found, might have called for certain adjustments in techniques. The factor analyses required a minimum of two tests for each known factor. The two best SI tests for each factor were used, except where it was possible to add to the battery a few scores that the school had on record, derived from standard tests of academic aptitude. An additional hour of testing time was taken up with administration of an appropriate achievement examination constructed for this study by the Aptitudes Project.

In accordance with the previous statement of preferences, most of the factors and their tests were in the symbolic category. For that reason, Table 2 has been constructed to help the reader associate the tests with the structure of intellect. The matrix shown in Table 2 represents all the factors in which symbolic information is the content. Symbolic content is in the form of letters and numbers, and other items commonly used for coding purposes. It is the stuff of which algebraic statements are made. The columns of the matrix represent the five kinds of operations and the rows represent the product categories, with the letter symbols as previously defined.

When the algebra study was initiated, only 16 of the 30 potential symbolic factors were thought to have been demonstrated, with some doubts regarding factor ESU. A previously found factor had been tentatively placed in that cell. The letter K stands for the abilities presumed to be known at the beginning of this study. Elsewhere, the letter I indicates a factor under investigation at the time and subsequently demonstrated in one or two analyses (Gershon, et al, 1963). The second letter, P, where it occurs, indicates a factor of primary concern in this investigation, which means that it was to be tested by factor analysis. The letter S, where it occurs, indicates a factor of secondary interest, in that no attempt was made to determine its validity as a factor variable in the populations sampled. The two divergent-production factors in this category each lacked two good tests for such a purpose. The three evaluation factors in this category are the subject of an investigation that was in the stage of test development, and tests that could be used were not ready until several months after the primary battery was administered. The late administration of a two-hour battery was devoted to tests of all the factors labeled S in Table 2. The results of this testing were available for predictive-validity study but not for construct-validity study. It is expected that later analyses will make possible a more dependable relating of their factors logically to the mathematics criteria, if their tests prove to be related.

It will be noted that the products of units and classes were entirely untouched by factor tests in this investigation. Concentration was heaviest in the rows for relations, systems, and implications. Representation of the factors of transformation abilities was weak because only one such factor had been demonstrated. Ideally, all the factors in the last four rows could well have been included.

Outside the area of symbolic abilities, only four factors were included, all from the semantic category. Factors CMU (verbal comprehension) and CMS (general reasoning) were included because, as stated earlier, they strongly represent the factorial contributions of verbal and quantitative academic-aptitude test batteries. In

addition there was the factor NMS (convergent production of semantic systems), the ability to organize a meaningful sequence or other kinds of system, with the thought that the algebra student who may be less able in symbolic abilities may organize his strategy in algebraic problem solving in terms of verbalized steps. Factor EMR (evaluation of semantic relations) had previously been best represented by a test of syllogistic reasoning, in which conclusions are tested. To the extent that the algebra student tests his conclusions by checking his verbally conceived relations of conclusions to premises, this factor could be of some importance in individual differences in achievement.

Table 2
A Matrix of the Structure-of-Intellect Abilities Pertaining
to Symbolic Information*

		Operations				
		C	M	D	N	E
Products	U	K	K	K		K?
	C	K		K		
	R	K, P	K	I, P	K, P	K, S
	S	K, P		I, S	K, P	S
	T				K, P	S
	I	K, P	K, P	I, S	K, P	S

*K: Factor known (previously demonstrated).
I: Factor under investigation in a concurrent study.
P: Factor of primary concern in this study.
S: Factor of secondary concern.

The Translation Problem in Mathematics. —We have emphasized the point that the kind of informational content of mathematics is primarily symbolic. It is often pointed out that mathematics is a special kind of language, with its own vocabulary and its own syntax and principles of expression. For the student of mathematics, particularly, there is a serious translation problem, since instruction, in textbook and in the classroom, is largely in terms of English. The student must not only learn discourse in a new language but must be able to go back and forth from the old to the new. Psychologically, the translation is from semantic to symbolic information; also in the reverse direction. At some time the student must also learn to apply algebra to the solution of verbally presented problems. Such problems definitely demand translation.

The logical problem for us at this point is whether the SI abilities are adequate to the task of accounting for the mastery of such translation skills or whether the process of translation demands additional, perhaps second-order, abilities of some kind. Present thinking on this problem leads us to the view that no extra-SI abilities are needed; that successful translation requires a certain minimum status in both symbolic and semantic abilities and that being high in both kinds ensures the potential necessary to achieve successful translation. Whether very high status in the one kind of ability can make up for lack of ability in the other is another question, partially answered by an earlier statement that a certain minimum status in both is believed to be essential; in other words, if there are compensatory features, there must be lower bounds to their operation.

The structuring of a word problem has been shown to depend heavily upon factor CMS (Kettner, et al, 1956). If the problem is very complex, factor NMS, or even DMS, might also become involved. The corresponding symbolic abilities are CSS, NSS, and DSS. In connection with the translation of problems, then, these six abilities should be investigated.

We have not attempted to solve this particular translation problem in this investigation. To investigate the problem thoroughly, one strategy might well introduce some new hypotheses regarding abilities over and above those in the SI model that might be involved in the translation operation. In the present investigation, we can only note where semantic abilities appear to be relevant, in addition to or in place of their corresponding symbolic abilities. Such instances will be noted in presenting the results.

Factors and Disciplinary Values of Algebra. —Educators, much more often in the remote past than in the recent past, commonly defended the teaching of algebra on the basis of its supposed disciplinary value in "teaching students how to think." The prevailing stimulus-response psychology left little room for belief in transfer effects of other than specific kinds. On the other hand, factor theory can provide a new basis for belief in disciplinary values, for we can conceive of the intellectual factors, including those pertinent to success in algebra, as somewhat general, thinking skills, subject to improvement with practice and transferrable, under appropriate conditions (Guilford, 1960b, 1961). Furthermore, a knowledge of the nature of those generalized skills gives us clues as to where they may be exercised and where and how they may be transferred. The transfer may be across content boundaries, for example, as implied in the discussion of the translation problem. This is not the place to elaborate upon this general conception of transfer, but it does give us further motivation for finding out more about the intellectual factors and their operations in connection with different school subjects.

THE APTITUDE AND ACHIEVEMENT TESTS

General Nature of the Tests

Factor Tests. —The strategy involved in designing factor-analytic studies necessitates the inclusion of a sufficient number of tests for each factor, so that each factor may be clearly and unambiguously defined. There are two conditions under which the number of tests employed to define a factor may differ: tests hypothesized to measure a previously undefined factor should number at least three, preferably four, so as to enhance the possibility that the factor is better understood in terms of the apparent psychological function shared by all its tests. At least two tests should be included for a factor that has been previously defined and where there is reasonable evidence that both tests are fairly pure measures of that factor.

When a study is concerned primarily with predictive validation, and a maximum of predictive variance is desired, it would be reasonable to include only one test of each factor thought to contribute to the prediction. When a greater depth of understanding is required with respect to what aptitudes are present, however, one predictor per factor will not suffice. When only one test thought to define a factor is employed in a validation study, there is no possibility of confirming that the factor is really present among the predictors. The investigator is then forced merely to assume that the factor found previously is the same as that which is implicitly contained within the predictor variables. The inclusion of at least two tests for each factor enables the investigator to determine whether the factor tests are functioning as expected in the particular validation study.

The factors chosen for investigation in this study had either (a) been found to be important in other studies of algebraic ability; or (b) were thought to be important as a result of a logical analysis of algebraic ability. The choice of those factors belonging to the second category was based primarily upon past research results of the Aptitudes Project. See Table 3 for a complete listing of all predictor tests utilized in all the aspects of this study.

Factors found important in other studies and hypothesized to be necessary for an accurate evaluation of ninth-grade algebra abilities were: CSR (symbolic relations), CSS (symbolic patterns), CMU (verbal comprehension), CMS (general reasoning), MSI (numerical facility), NSR (symbolic correlates), NSS (convergent production of symbolic systems), NMS (ordering), and EMR (logical evaluation). These factors have already been mentioned as expected predictors of algebraic abilities and are included in Table 1.

Other factors not identified in the reference studies cited above, but thought to be important in algebraic ability, include: CSI (cognition of symbolic implications), DSR (divergent production of symbolic relations), and NST (symbolic redefinition). All of the aforementioned factors were represented by tests in the factor analyses.

Consistent with the logical analysis of SI abilities necessary for algebraic aptitude, several tests of factors concerned with the evaluation and the divergent production of symbolic information were included in an additional prediction battery. Tests of these additional factors, developed for use in a concurrent investigation at the Aptitudes Project, were given at a date later than the factor-analytic battery and were not subjected to factor analysis. The factors represented in the additional battery were hypothesized to be: DSS (divergent production of symbolic systems), DSI (divergent production of symbolic implications), ESR (evaluation of symbolic relations), ESS (evaluation of symbolic systems), EST (evaluation of symbolic transformations), and ESI (evaluation of symbolic implications). Testing-time commitments necessitated the short length of this battery and made it impossible to include enough tests to support the factor analysis of its tests.

Standard Tests. —Additionally, test-score information for several standard academic-aptitude measures was obtained from the school-district records. Since these tests typically supply the school with prediction information, they were included in this study in order to compare their efficiency with that obtained from the factor tests. The tests were: California Test of Mental Maturity (CTMM), Differential Aptitude Tests (DAT), and Iowa Test of Basic Skills (Iowa). As will be seen later, not all of these tests were utilized in the prediction studies, i. e., not all subtest scores were considered logically relevant. The school measures that were included in the factor analyses were: CTMM Language MA (for factor CMU), Iowa Reading Comprehension (CMU), and DAT Numerical Ability (MSI). The verbal-comprehension (CMU) tests were included in the factor analysis so that the possible CMU variance in any test could be relegated to its appropriate factor and not left to bias the factorial

Table 3
List of Tests and Factors They Were Intended to Represent

Tests included in the factor battery and the prediction studies	Structure-of-intellect factors																			
	C S R	C S S	C S I	C M U	C M S	M S I	D S R	D S S	D S I	N S R	N S S	N S T	N S I	N M S	E S R	E S S	E S T	E S I	E M R	
1. Alternate Additions - DSR01B							x													
2. Best Trend Name - EMR01A																			x	
3. Camouflaged Words - NST01A													x							
4. Circle Reasoning - CSS01C		x																		
5. Correlate Completion II - NSR01A										x										
6. Form Reasoning - NSI02B													x							
7. Letter-Number - NSI03A													x							
8. Letter Series - NSR02B										x										
9. Letter Triangle - CSS02B		x																		
10. Matched Verbal Relations - EMR02A																			x	
11. Necessary Facts - CMS04A					x															
12. Number Rules - DSR02B							x													
13. Numerical Operations - MSI01A						x														
14. Picture Arrangement - NMS01B														x						
15. Right Order Test - NSS01B											x									
16. Seeing Trends II - CSR01A	x																			
17. Sentence Order - NMS03A															x					
18. Ship Destination Test - Form A-2 (CMS)					x															
19. Sign Changes - NSI01A													x							
20. Symbol Grouping - CSI01A			x																	
21. Word Changes - NSS02B											x									
22. Word Linkage - EMR03A																			x	
23. Word Patterns - CSI03B			x																	
24. Word Relations - CSR02B	x																			
25. Word Transformation - NST02A												x								
26. CTMM Language MA (CMU)				x																
28. Iowa Reading Comprehension (CMU)				x																
32. DAT Numerical Ability (MSI)						x														
Tests per factor	2	2	2	2	2	2	2	0	0	2	2	2	3	2	0	0	0	0	3	
Factor tests included in the prediction battery but not in the factor analysis																				
38. Abbreviations - ESI01A																		x		
39. Condensations - EST01A																	x			
40. Letter-Number Scales - ESS01A															x					
41. Limited Words - DSI02B									x											
42. Most Similar Sets - ESS02A																x				
43. Number Combinations - DSS02B								x												
44. Sign Changes II - ESR01B															x					
Tests per factor	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1	2	1	1	0	
Standard-test measures included in the prediction studies but not in the factor analysis, and their assumed dominant factors																				
27. CTMM Non-Language MA (CMU & MSI)																				
29. Iowa Arithmetic Concepts - Test A-1 (CMU)																				
30. Iowa Arithmetic Problem Solving - Test A-2 (CMU & MSI)																				
31. DAT Verbal Reasoning (CMU, CMR, & NMR)																				
33. DAT Abstract Reasoning (CFR)																				
34. DAT Clerical Speed and Accuracy (ESU)																				

picture of the test battery. Since CMU is an easily demonstrated factor, two tests were thought sufficient to define it. Likewise, the MSI test was included in the analysis to help define factor MSI, which was hypothesized for only one factor test, Numerical Operations.

Achievement Tests. — The achievement tests were specifically designed to assess student accomplishment of objectives as stated by the school mathematics department. The general objectives, which served as guides in the construction of these tests, are as follows:

Basic Mathematics: Increasing the competence of the student in fundamental operations of arithmetic and in the use of basic formulas.

Non-college Algebra: Further increasing the competence of the student in arithmetic operations and introducing basic algebraic and geometric concepts for those pupils needing additional preparation for entering the regular, college-preparatory mathematics program.

Regular Algebra: Development of basic algebraic skills and concepts.

Accelerated Algebra: Review of Regular Algebra and continuation through intermediate algebra.

The general-mathematics test structure was such that it could be utilized as a measure of achievement in both Basic-Mathematics and Non-college-Algebra course work. The algebra test was constructed as a measure of achievement for both Regular and Accelerated Algebra.

Working with the mathematics department, and utilizing the general course objectives as basic reference points, specific course objectives were derived which served as a basis for choosing the types of problems to be included in the achievement tests. A careful analysis of the subject matter to be included indicated that a multiple-choice format would be quite adequate for testing the understanding of certain concepts and skills. A completion format was utilized for problems of a more complex nature. The achievement tests were designed so that each could be administered during one class period.

Specific Nature of the Tests

The Factor Tests. — The tests designed for each factor included in the factor-analyzed battery are listed below.² The reader will find the test information organized as follows: (a) the SI trigram symbol; (b) the complete, formal SI name; (c) the popular factor name (where one exists); (d) a brief description of the general intellectual task involved; (e) an alphabetized list of the employed tests hypothesized for that factor, along with their identification numbers for this study; and (f) a note regarding the population that had been used in prior studies in which each particular test had been investigated, i.e., "A" for adults and "C" for children. The number following the "C" denotes the grade level tested. Where a test had been developed for this study it is classified as "New." The factor tests are more completely described in Appendix A.

CSR - cognition of symbolic relations (symbolic relations). Recognition of the relationship between symbolic units.

- 16. Seeing Trends II - CSR01A (A)
- 24. Word Relations - CSR02B (A)³

CSS - cognition of symbolic systems (symbolic patterns). Awareness of a symbolic system organized according to specific rules.

- 4. Circle Reasoning - CSS01C (A)
- 9. Letter Triangle - CSS02B (A)^{*}

CSI - cognition of symbolic implications. Discovery or awareness of symbolic consequents.

- 20. Symbol Grouping - CSI01A (A)^{*}
- 23. Word Patterns - CSI03B (A)^{*}

CMU - cognition of semantic units (verbal comprehension). Awareness of the meaning of words and ideas.

- 26. CTMM Language MA - CMU
- 28. Iowa Reading Comprehension - CMU

CMS - cognition of semantic systems (general reasoning). Awareness or recognition of the nature of a problem.

- 11. Necessary Facts - CMS04A (A)^{*}
- 18. Ship Destination Test - Form A-2 - CMS (A)^{*}

MSI - memory for symbolic implications (numerical facility). Long-term memory for the manipulation of symbols according to a given set of rules.

- 13. Numerical Operations - MSI01A (C-9)
- 32. DAT Numerical Ability - MSI

² All of the factor tests had been originally developed under Contracts N6onr-23810 and Nonr-228(20) with the Office of Naval Research.

³ Starred tests were revised to some degree to make them more appropriate for the ninth-grade population. Typically, the revisions were restricted to the instruction page.

- DSR - divergent production of symbolic relations. Fluency in the production of symbolic relations.
 1. Alternate Additions - DSR01B (C-9)
 12. Number Rules - DSR02B (C-9)
- NSR - convergent production of symbolic relations (symbolic correlates). Convergently completing a symbolic relationship based upon a given set of conditions.
 5. Correlate Completion II - NSR01A (A)
 8. Letter Series - NSR02B (A)
- NSS - convergent production of symbolic systems. Convergently producing a system starting with a few symbolic units.
 15. Right Order Test - NSS01B (A)
 21. Word Changes - NSS02B (A)*
- NST - convergent production of symbolic transformations (symbolic redefinition). Convergently producing new symbolic units from a given set according to specific rules.
 3. Camouflaged Words - NST01A (A)*
 25. Word Transformation - NST03A (A)*
- NSI - convergent production of symbolic implications. Convergently producing symbolic consequents based upon a given system of symbolic relations.
 6. Form Reasoning - NSI02B (A)*
 7. Letter Number - NSI03A (New)
 19. Sign Changes - NSI01A (A)*
- NMS - convergent production of semantic systems (ordering). Convergently producing a logical system (order) from a set of scrambled ideas.
 14. Picture Arrangement - NMS01B (A)*
 17. Sentence Order - NMS03A (A)
- EMR - evaluation of semantic relations (logical evaluation). Decision making based upon a consideration of the more crucial relationships inherent in the given set of meaningful units.⁴
 2. Best Trend Name - EMR01A (New)
 10. Matched Verbal Relations - EMR02A (New)
 22. Word Linkage - EMR03A (New)

Tests designed for the factors included in the additional short prediction battery were more experimental. Most of them had never been factor analyzed and the rationale for using them is to be found in the logical analysis of the intellectual abilities thought necessary for successful handling of algebra. The content of all these tests is symbolic and the operations include divergent production and evaluation. The four products involved are relations, systems, transformations, and implications. The tests included in the prediction battery and their expected factor content were (by factor):

- DSS - divergent production of symbolic systems. Divergently producing systems of symbols, e.g., equations.
 43. Number Combinations - DSS02B
- DSI - divergent production of symbolic implications. Divergently producing consequents of symbolic material.
 41. Limited Words - DSI02B
- ESR - evaluation of symbolic relations. Sensitivity to the accuracy of stated relations between symbols.
 44. Sign Changes II - ESR01B
- ESS - evaluation of symbolic systems. Sensitivity to accuracy of sets of inter-relationships among symbols.
 40. Letter-Number Scales - ESS01A
 42. Most Similar Sets - ESS02A
- EST - evaluation of symbolic transformations. Sensitivity to adequacy of slight changes in symbolic material.
 39. Condensations - EST01A
- ESI - evaluation of symbolic implications. Sensitivity to adequacy of consequents of symbolic content.
 38. Abbreviations - ESI01A

The Achievement Tests. — The content of the achievement tests was derived from an analysis of the specific objectives for each of the four courses. (See Table 4 for specific course objectives.) The general procedure followed was that of choosing the category of problem, the correct solution of which would indicate achievement of the related objective. Because of the limited time available for administering the achievement tests, it was necessary to use extreme care in choosing problems. The objectives for Basic Mathematics and Non-college Algebra were combined and the problems designed to test for their achievement were compiled in constructing the General Mathematics (G. M.) Achievement Test.

⁴ These EMR tests have been more thoroughly investigated with an account to be given in Reports from the Psychological Laboratory, No. 32 (in preparation).

Table 4
Ninth-Grade Course Objectives

Course	Specific Objectives	Representative Problems	Course Text Book
Basic Mathematics	1. Improvement of ability in arithmetic fundamentals.	1. Problems involving fundamental operations which utilize whole numbers, fractions, mixed numbers, and decimal fractions.	Peters, M., 1956.
Non-college Algebra	1. Improvement of ability in arithmetic fundamentals. 2. Knowledge of some measurement principles. 3. Knowledge and use of formulas. 4. Knowledge and use of informal geometry. 5. Knowledge and use of graphs and tables. 6. Knowledge of fundamental algebraic operations.	1. Same as for Basic Mathematics. 2. Rounding significant digits and interpreting scaled drawings. 3. Calculating simple interest. 4. Learning plane geometric figures and deriving the formulas for the calculation of their measurements. 5. Interpreting line graphs and use of the square-root table. 6. Simple problems involving the elementary use of axioms, signed numbers, literal numbers, exponents, etc.	Gager, W.A., 1953.
Regular Algebra (Algebra I)	1. Knowledge of fundamental algebraic operations. 2. Knowledge and application of first, second, and third degree equations. 3. Knowledge and application of special products and factoring. 4. Knowledge and application of algebraic expressions utilizing fractions.	1. Same as No. 6 for Non-college Algebra, but with greater variety and complexity. 2. Solving first, second, and third degree equations involving the use of literal, integer, fractional, or decimal coefficients. 3. Solving word problems involving the use of the equations and coefficients noted in 2. 4. Computing the products of monomials and binomials utilizing short cues; factoring perfect square trinomials, the difference of two squares, quadratic trinomials and the sums and differences of two cubes. 5. Reducing fractional expressions to lowest terms; simplifying mixed expressions and complex fractions; solving fractional equations; solving word problems involving fractions, i.e., ratio and proportion problems.	Shute, W.G., 1960.
Accelerated Algebra	1. Greater proficiency in meeting the specific objectives stated for Regular Algebra.	1. Same as for Regular Algebra but involving greater variety and complexity.	Hills, J.E., and Mazziotta, Estelle, 1959.

The first section of the G. M. test was composed of 10 multiple-choice problems, each containing three alternatives. The balance of the test was composed of 20 completion problems. Students were instructed to show all their work. The following is a list of the types of problems contained in the two sections of the G. M. test.

Multiple-choice problems

1. Derivation of the rectangle perimeter formula.
2. Computation of simple interest.
3. Simple trigonometric expression of the tangent relationship for a right triangle.
4. The use of the tangent relationship in indirect measurement.
5. The use of the Pythagorean relationship in the solution of problems involving right-triangle dimensions.
6. Multiplication and division of literal numbers, involving exponents.
7. Solution of triangle dimensions where the triangles are similar.
8. Solution of first-degree equations, one involving a proportional set-up which makes use of the means-extremes relationship.

Completion problems

1. Addition, subtraction, multiplication, and division of positive integers, some involving the use of decimals.
2. Addition, subtraction, multiplication, and division of fractions.
3. Addition, subtraction, multiplication, and division of mixed numbers.
4. Order of operations.
5. Addition, subtraction, multiplication, and division of signed numbers.
6. The use of ratio representation in scale drawing.
7. Reading graphs.
8. Facility in using the square-root table.

Since the general objectives for Regular Algebra and Accelerated Algebra were essentially the same, the two groups were combined into one larger group which is henceforth referred to as the algebra group. The achievement test for the algebra group was structured similarly to the general-mathematics-achievement test. The first section contained 16 multiple-choice items, each with three alternatives. The second section was composed of 19 completion problems. Again emphasis was placed upon the students showing all their work. The types of problems contained within each section are as follows:

Multiple-choice problems

1. Simplifying algebraic expressions.
2. Multiplication and division of monomials.
3. Numerical evaluation of a literal statement through substitution.
4. Simplification of exponential expressions.
5. Cube root of algebraic expressions.
6. Solution of first-degree equations, literal numbers and fractional terms.
7. Reduction of algebraic fractions to lowest terms.
8. Solution of second-degree equations.
9. Identification of a $4 \times 5 \times 7$ triangle as acute.
10. Derivation of the total surface area formula for a rectangular solid.
11. Identification of types of angles found in overlapping triangles.
12. Accurate recognition of the correct equation for representing direct variation.

Completion problems

1. Addition, subtraction, multiplication, and division of polynomials.
2. Translation of a partly semantic, partly symbolic algebraic statement into an equivalent, completely symbolic statement.
3. Finding the square root of a fractional, algebraic expression.
4. Solution of a fractional, quadratic equation.
5. Finding the product of two binomials.
6. Factoring the difference of two squares.
7. Factoring both the sum and difference of two cubes.
8. Multiplication and division of monomial fractions.
9. Factoring quadratic trinomials.
10. Simplifying a complex fraction involving a binomial numerator and denominator.
11. Solution of word problems involving ratios and simultaneous equations.

PROCEDURES

General Strategy

The discussion thus far has indicated some of the early major steps involved in this investigation, including the study of previous reports of investigations of similar nature, the logical consideration of SI factors possibly involved in the learning of general mathematics and algebra, and the planning of the predictor tests and achievement examinations. Other major steps involved conferring with the school district in which the testing was to be done, with respect to the nature of the courses in ninth-grade mathematics, with respect to school records that could be utilized, and with respect to testing arrangements. Finally, there were the data-treatment operations, including the factor analyses to be performed, the preparation of the criteria information, and the multiple-regression studies of the data. The later steps will now be described in some detail.

Specific Steps

Administration of the Aptitude Tests. — The factor battery of 25 tests designed to be factor analyzed was administered on Tuesday, Wednesday, and Thursday, of the second week of October, 1961.⁵ The tests were administered by Project personnel, with the school counseling department providing additional proctors. The entire ninth grade of approximately 600 pupils was tested in the school gymnasium. The booklets and their included tests were administered in the following order:

Tuesday, October 10, 1961: Booklets A and B with the following tests: Letter Series, Picture Arrangement, Word Patterns, Form Reasoning, Circle Reasoning, Matched Verbal Relations, Word Relations, Camouflaged Words, and Sentence Order.

Wednesday, October 11, 1961: Booklets C and D with the following tests: Alternate Additions, Word Linkage, Right Order Test, Ship Destination Test, Seeing Trends II, Sign Changes, Best Trend Name, and Letter Triangle.

Thursday, October 12, 1961: Booklets E and F with the following tests: Word Changes, Necessary Facts, Correlate Completion II, Letter-Number, Number Rules, Symbol Grouping, Word Transformation, and Numerical Operations.

The additional predictor battery was administered during May of 1962. The testing was done in the mathematics classes, each consisting of approximately 35 pupils. Project personnel administered the tests with the class teachers proctoring. This battery consisted of two booklets and was administered as follows:

⁵ We are very much indebted to the Lynwood, California, school district, which generously gave testing time, and to the teachers and students who participated, for their cooperation.

Wednesday, May 2, 1962: Booklet G with the following tests: Number Combinations, Abbreviations, Letter-Number Scales, and Condensations.

Thursday, May 3, 1962: Booklet H with the following tests: Sign Changes II, Most Similar Sets, and Limited Words.

The testing conditions under which the main factor battery was administered were such that a few students seemed restless and occasionally inattentive. For the most part, however, the majority of the approximately 600 subjects listened to the reading of the instructions over the loud speaker system and followed along reading silently. The actual test taking was relatively smooth; the exception being where a student was faced with a test which was, for him, quite difficult. Restlessness in this case was to be expected.

The testing conditions for the additional predictor battery were better, in that it was given to small groups of students in their mathematics classes. Here, supervision improved. Over all, it can be said that no special problems evolved which may have had significant influence upon the results of either test battery.

All tests were scored completely objectively except a few in which some subjective judgment was needed, including: Alternate Additions, Seeing Trends II, Necessary Facts, Number Rules, and Symbol Grouping. Judgments had been fairly well standardized in advance.

The Standard Tests.—All but a few of the ninth-grade students had been administered the CTMM, the DAT, and the Iowa Tests when they were in the eighth grade. Those who had missed the eighth-grade testing were administered the tests at the beginning of the ninth grade, during the fall of 1961. For such students, minor adjustments were made in scores to make them more comparable with what they would probably have obtained in the previous school year.

Owing to the shortage of tests in the battery designed for factor analysis, three part scores from the standard tests were borrowed to serve as marker tests for factors CMU and MSI (verbal comprehension and numerical facility). This practice was kept to a minimum, since it was realized that most of the standard-test scores are factorially complex. The tests utilized in the factor analysis are indicated in Table 2.

The Achievement Tests.—The general-mathematics-achievement test was administered on Wednesday, May 23, 1962, and the algebra-achievement test on Thursday, May 24, the day following. These tests were administered in the regular class sections, with highly satisfactory testing conditions prevailing. The achievement-test scores used were obtained near the end of a full year of instruction in the courses so as to take advantage of as much learning as possible in the courses.

Two methods were used in scoring the achievement tests. One was the traditional count of the number of problems correctly solved; the other was called "construct scoring." The reason for this title is that an attempt was made to score performances more analytically along the lines of the factors from the structure of intellect. For this purpose, every problem in the test was examined carefully, noting at what points in the process of norms' solution of the problem there could be failure. The ability that seemed most crucial for the step in question was given a point credit if the step was done correctly; no credit if not done correctly. The construct-scoring method is explained in the following problems and their solutions:

Example I:

Divide: $(3x^2 - 17x - 6)$ by $(x - 6)$

$$\begin{array}{r} 3x + 1 \\ x - 6 \overline{) 3x^2 - 17x - 6} \\ \underline{3x^2 - 18x} \\ x - 6 \\ \underline{x - 6} \\ 0 \end{array}$$

Explanation:

- a. Traditional scoring would give the student one point if correct and no point if wrong.
- b. Construct scoring would yield the following point credit:
 1. All arithmetic correct: 1 MSI point.
 2. Correct division of literal coefficients: 1 NSR point.
 3. Correct combinations of literal coefficients: 1 NSC point.
 4. Correct multiplication of literal coefficients: 1 NSR point.

Example II:

Solve: If it takes Mary 10 hours to type a report and it takes Barbara 15 hours to do the same job, how long will it take them to type the report together? Show all work.

Let x = hours required to type report together

Let $\frac{1}{10}x$ = proportion done by Mary

Let $\frac{1}{15}x$ = proportion done by Barbara

$$\begin{array}{rcl} \frac{1}{10}x + \frac{1}{15}x + 1 & & 3x + 2x = 30 \\ & & 5x = 30 \\ & & x = 6 \end{array}$$

Explanation:

- a. Traditional scoring would give the student one point if correct and no point if wrong.
- b. Construct scoring would yield the following point credit:
 1. All arithmetic correct: 1 MSI point.
 2. Correct representation of the proportions of work done by Mary and Barbara: 1 NSR point.
 3. Correct equation set-up: 1 CMS point and 1 NSS point.
 4. Correct conversion of fractions to whole number: 1 CSC point and 1 NSU point.
 5. Correct combination of like terms: 1 NSU point.
 6. Correct usage of the division axiom: 1 NST point.

The concept of construct scoring seemed to be quite sound and potentially useful. If the dominant intellectual skill hypothesized to be crucial for success or failure at any point in the solution of a problem is a sound guess, a collection of score points thus obtained for a factor from a series of problems should approach a measurement of that factor as it applies to that kind of operation in problem solving. In everyday life, a teacher might thus pinpoint areas of intellectual strengths and weaknesses for students and for classes of students. In the present investigation, we should find that each factor score thus obtained should serve as a different criterion for prediction from tests of that same factor.

Unfortunately, things did not work out as expected. The respective factor scores derived from the achievement tests did not correlate generally highest with aptitude-test scores of the same factors. Although some of the construct scores intercorrelated near zero (the lowest r was .01), most of the intercorrelations were of substantial size, the highest being .85. The reason is that, in spite of care in applying the procedures of construct scoring, there is much experimental interdependence among the scores. This was particularly true in problem solutions in which a few steps are completed but the examinee does not do later steps. In some instances there is contingent dependency. For example, suppose that the first step is that of combining literal coefficients by reason of their similarity (NSC), and the next step is addition (MSI). If the first step is done correctly, the second may be done either correctly or incorrectly. If the first step has to be accomplished correctly before the second can be done at all, there is interdependence. In other instances, a certain step may be done incorrectly and the immediately following step may be done anyway, either correctly or incorrectly. With more than two steps, the inter-relationships and contingencies become quite complicated. It might be possible to consider the number of opportunities to solve a certain step, regardless of whatever else has happened, and to derive percentage scores of correct performance of each kind. We did not have time to try out this much refinement. The outcome was that we simply summed all score points from construct scoring, hoping that such a criterion might be more predictable than that from traditional scoring.

Grade-point Criteria. — To provide another criterion, second-semester grades assigned by the teachers were obtained from the school, using the grade-point equivalent of the letter-grade mean. The means and variances of grade-point scores were calculated for each mathematics teacher (for each homogeneous group). A homogeneous group consisted of all mathematics students of the same level, e.g., Regular Algebra, for each teacher. Statistical analysis of the grades, utilizing analysis of variance and t tests, indicated that the grades assigned by some teachers were not homogeneous with those assigned by other teachers. In view of the heterogeneity found, each teacher's grades for each homogeneous group were dichotomized as near the teacher's median as possible and scores of 0 and 1 were assigned. Actually, because of the coarseness of the grading scale, only a very few distributions were dichotomized at different places than those of the majority. Students within each of the four mathematics courses were then combined to give four criterion samples for validation study.

Since the first-order validity coefficients were in the nature of point-biserial correlation coefficients, the beta coefficients and the multiple R 's were also smaller than such statistics would be when computed from the use of Pearson r 's. It can be shown that a simple ratio of $\sqrt{pq}/y$ can be applied to both the betas and the R 's in order to bring the results into line with those found by using Pearson r 's throughout. This procedure was followed.

The Factor Analyses. — For purposes of factor analyzing the predictor tests, four overlapping groups of subjects were utilized: (1) the general-mathematics group, composed of basic-mathematics and non-college-algebra students; (2) the algebra group, composed of regular- and accelerated-algebra students; (3) boys; and (4) girls.⁶ The number of subjects in each of the four groups was, for general mathematics, 217; for algebra, 211; for boys, 201; and for girls, 227.

Because the uses of the results of the four factor analyses were to be different, two different procedures were employed. The factor-analyzed variables for the general-mathematics and the algebra groups consisted of all 25 factor tests and 3 subtests of standard tests. These 28 variables are the first listed in Table 3. The goals of these two factor analyses were two: the reconfirmation of factor content of experimental factor tests in these populations, and the determination of factor content of each test so that it might be appropriately weighted in deriving its factor score in the prediction aspect of this study. For these reasons, all 28 variables were employed.

The goal of the factor analyses in the boys and the girls groups was simply to compare the factor structure of abilities for the two sexes, to determine whether the combining of the sexes in factor analyses could be justified. Since there was some reason to suspect complexity of the standard subtest scores, only the first 25 variables were factor analyzed. The comparison of factor structure over sex, then, was made including only the factors emerging from the SI factor tests.

Factors were extracted by means of the principal-components method. A study of the factor matrices for the general-mathematics and the algebra groups indicated that computer rotations for the first 14 factors would likely prove fruitful. The varimax method was utilized for the machine rotations. The varimax rotations for the algebra and general-mathematics groups proved to be inconclusive as an end product, so further rotations were carried out utilizing Zimmerman's graphical method (Zimmerman, 1946). Using the criteria of positive manifold, simple structure, and psychological meaningfulness, final solutions for both groups were reached.

The comparison of factor structures between the sexes was made from the twelve varimax-rotated factors obtained from the computer. No graphic rotations were employed, for it was believed that the comparison was best made between entirely objectively rotated solutions, and such a comparison would answer the one question regarding similarity of factor structures. The rotations in the two mathematics groups, however, had been

⁶ The factor-analytic and related computations were done on the IBM 7090 computer at the Western Data Processing Center at the University of California at Los Angeles.

aimed at achieving the most meaningful factors in the two cases. The added freedom offered in graphic rotations might be expected to alter similarity of structure. In both cases, however, a statistical test was applied in order to derive more definite information concerning similarity of factor structure, as between the two mathematical groups as well as between the two sex groups. Tucker's coefficient of congruence, ϕ_T , was used as the comparison statistic (Tucker, 1951).

The relative importance of the different factors in the mathematics criteria could be observed by taking several different approaches. A criterion could have been factor analyzed along with the factor tests, but such a variable was not included in the factor-analytic battery, since we did not wish to run the risk of confusing the factor picture of the tests by including a very complex variable, especially with a limited number of marker tests to represent their factors in the battery. It is possible to extend an analysis already completed to an additional variable, but no computer program was known to exist for this kind of operation. Thus, the information used regarding the relative importance of the factors in the criteria was derived from the standard, partial regression weights (beta weights) in the multiple-regression analyses. This procedure will be defended when results are discussed.

Determination of Factor Scores.—There were several reasons for deriving factor scores from the tests for use in multiple-regression analysis. One reason is that this would give a clearer view of the importance of each factor than could be obtained by using tests separately. Factor scores, where derived from two or more tests, are stronger measures of factors than are single tests. The number of predictors is reduced, thus reducing shrinkage in estimating unbiased multiple correlations. Another compelling reason is that factor scores give lower intercorrelations among the predictors, a condition favorable for higher multiple R's. Related to the latter reason is the fact that this procedure, if properly carried out, removes all linear restraints in the multiple-prediction situation.

As recommended by Thurstone (1947) and by Harman (1960), the complete determination of factor scores involves complex operations. Traditionally, since such operations have not been programmed for computer solutions, some simpler, substitute method has been employed. The method used in this study simply summed the standard scores (z scores) of the tests uniquely selected for each factor. A computer program was available for these operations.

Such approximations to factor scores have one disadvantage. Because the tests are usually not completely univocal for a factor, and no suppression of secondary variances is achieved by this simple summing procedure, the factor scores are likely to be correlated, in spite of the fact that the factors that they represent may be uncorrelated. The intercorrelations of factor scores, each derived from two or more tests, should be generally lower, however, than the intercorrelations among single tests. And, of course, there were some opportunities for suppression effects to occur in the normal processes of deriving multiple-regression equations. The highest intercorrelation of factor scores actually obtained was .50.

Because of certain distinct differences between the factor structures derived from the two course groups, there were some resulting differences in the combinations of tests needed to obtain factor scores in the general-mathematics and algebra populations. Table 5 provides a ready comparison of the component tests for deriving the scores for each factor in the two populations. In only four cases of the 13 were the sources of factor scores identical for the two populations, but in every case there was at least one test in common to the two.

Table 5
Factor Score Compositions

Variables contributing to factor score	Factor Score												
	CSR	CSS	CSI	CMU	CMS	MSI	DSR	NSR	NSS	NST	NSI	NMS	EMR
General- Mathematics Group	9	4	23	22	11	13	1	5	21	3	19	17	2
	16			26	18	32	12	8		6			10
					20			24		7			14
Algebra Group	16	4	6	11	18	13	1	5	15	3	19	14	2
	24		23	26	20	32	12	7	21	25		17	10
								8					22
								9					

Validation Procedures.—The information concerning test- and factor-score validity was obtained by correlating the test scores and factor scores with the various criteria of performance in the general-mathematics and algebra courses. Selected multiple-regression equations and multiple-correlation coefficients were computed so as to answer the following specific questions:

1. What is the relative predictive involvement of each factor (factor scores and factor tests) in connection with the four levels of ninth-grade mathematics courses, with achievement-test scores and with course grades as criteria?
2. How well does an optimally weighted battery of standard tests predict achievement in ninth-grade mathematics courses as compared with a battery of factor scores?

3. How much would a battery of optimally weighted factor scores improve prediction of mathematics achievement available from weighted combinations of tests in each of three standard aptitude batteries? Are the differences statistically significant?

4. How predictable are course grades as compared with scores on achievement examinations when predicted from a battery of factor scores vs. a battery of standard aptitude tests?

5. How predictable is a construct-score achievement criterion as compared with a traditionally scored achievement criterion?

In order to answer all these questions, and related minor ones, and to derive implications, it was necessary to carry out the computation of 56 multiple-regression equations. Table 6 lists the descriptions of those regression equations needed to answer the specific problems of this study. The different combinations of predictors, for purposes of comparison, are listed at the left of Table 6. The three criterion measures for each course group are given at the top of the table. Each number in the body of the table denotes a multiple-regression equation to be computed for each pertinent group. For example, the number 1 denotes a regression equation predicting the general-mathematics, traditionally scored, criterion-test score from the 13 factor scores. Since the general-mathematics group consists of two sub-groups with different course objectives and criteria of success, the number 1 refers to two such equations, one for the basic-mathematics students and the other for the non-college-algebra students.

Table 6
Descriptions of Multiple-Regression Equations
with their Predictors and Criteria

CRITERIA PREDICTORS						
	G. M. Criterion Test, Traditional Score	Algebra Criterion Test, Traditional Score	G. M. Grades	Algebra Grades	G. M. Total Construct Score	Algebra Total Construct Score
1. Aptitude Booklets A-F (13 factor scores)	1	11				
2. Aptitude Booklets A-H (13 factor scores and 7 test scores)	2	12	21	23	25	27
3. Aptitude Booklets G-H (7 test scores)	3	13				
4. Selected CTMM, DAT, & Iowa Measures (9 subtest scores)	4	14	22	24	26	28
5. CTMM (Lang. & Non-Lang.)	5	15				
6. A-F & CTMM (Lang. & Non-Lang.)	6	16				
7. DAT Composite	7	17				
8. A-F & DAT Composite	8	18				
9. Iowa Composite	9	19				
10. A-F & Iowa Composite	10	20				

Nature of the Populations and the Courses

The population of ninth-grade students represented in this study was drawn from a completely urban community which included no minority groups of any consequence. Skilled and unskilled labor comprise the major parental occupations. Major sources of income include manufacturing, service industries, wholesale and retail activities, and construction.

The high school is considered typical, and had an average total enrollment of about 2,100 pupils, the male-female ratio of which was approximately unity. The mathematics sections are considered to be typical with the exception of the accelerated groups. The ninth-grade mathematics curriculum consisted of four levels: Basic Mathematics, Non-college Algebra, Regular Algebra, and Accelerated Algebra. Table 7 indicates the size, the chronological-age, and the mental-age information for the four levels of those students employed in at least one aspect of this study.

Table 7

Demographic Information of the Samples

Levels	CA	MA	N
Basic Mathematics	13.8	12.8	109
Non-college Algebra	13.6	13.9	124
Regular Algebra	13.6	14.6	136
Accelerated Algebra	13.5	15.7	78

The four different levels of mathematical sophistication, in the form of four courses, provided the bases for the statement of four sets of specific objectives, all dependent to some extent upon the general objective set forth for work in arithmetic up through the seventh and eighth grades, i. e., arithmetic fundamentals. Table 3 describes the course levels from most simple to most difficult along with their corresponding objectives. Appropriate problems for the assessment of achievement of these objectives are noted. The courses were traditionally taught, by the lecture-demonstration-examination method. Students met every day at the same hour, in the same room, and with the same teacher.

RESULTS

Test Statistics

Means, Variances, and Reliabilities.⁷ — The data in Table 8 provide the descriptive statistics of most of the test variables used in this study. The first column lists the 44 test variables employed in at least one aspect of this study. To the left of the test names are variable numbers which remain constant throughout this report for all groups and all studies. The first pair of data columns describes the distribution of raw test scores. Distributions that were severely skewed, making the Pearson r an inappropriate index of correlation, were scaled to a normal distribution when that was possible; otherwise, the scores were dichotomized near the median. For the general-mathematics group, six test scores were dichotomized and nine were C-scaled into normally distributed scores. Only five score distributions were dichotomized for the algebra group and four were C-scaled. In general, most of the test-score distributions were fairly normal. Scores for the algebra group were more often normally distributed than for the general-mathematics group.

The next two pairs of data columns in Table 8 list the means and standard deviations of the raw scores. These statistics were not computed from binary scores, since means and standard deviations would both be near .50 and no comparison between groups could be made. Likewise, C-scaled scores would all exhibit means near 5.0 and standard deviations near 2.0. For every test, the mean score obtained by the algebra group is higher than that obtained by the general-mathematics group. The four empty cells in these columns are due to the fact that the general-mathematics group was not administered the algebra-achievement test and the algebra group was not administered the general-mathematics-achievement test.

The last pair of data columns in Table 8 lists reliability estimates of the tests. Unless specifically noted, the reliability coefficient is the Spearman-Brown estimate based upon the intercorrelation of separately timed part scores. Because some of the one-part tests were speeded tests, the Kuder-Richardson estimate of internal consistency reliability could not be made. In such cases, the communalities from the factor-analytic solutions are used as lower-bound estimates of reliability. Where test scores were dichotomized, the part scores were also dichotomized, and the reliability estimate reported is the Spearman-Brown estimate based upon the tetrachoric inter-part correlations.

The reliability estimates of the published tests are only reported when the estimate is an obtained communality estimate. Several estimates of different kinds of reliability can be found in the publishers' manuals for those tests.

Just as there is no significant comparison to be made between the two groups of students on the basis of test-score standard deviations, there is none to be made on the basis of reliability estimates. The reliability estimates of this study are consistent with those of previous studies with regard to tests shared by both.

Intercorrelations. — The intercorrelations of the test variables are given in Tables 9 and 10, for the general-mathematics and algebra groups, respectively. As in most aptitude-measure correlation matrices, the coefficients are predominantly positive and low in value, with very few negative coefficients. The sex-variable correlations are the only exceptions to this rule where a negative correlation means that girls had a higher mean in the test. All the coefficients in Tables 9 and 10, whether concerned with dichotomous or scaled scores, are

⁷ Shirley Peeke, research assistant, deserves a large share of the credit for the computer work involved in this analysis.

Table 8

Distribution, Means, Standard Deviations, and Reliabilities of Scores

TEST	FORMS OF DISTRIBUTION ³			MEAN		ST. DEV.		RELIABILITY	
	G. M.	ALG.		G. M.	ALG.	G. M.	ALG.	G. M.	ALG.
1 Alternate Additions - DSR01B	0	0		15.88	20.02	4.59	4.65	.81	.76
2 Best Trend Name - EMR01A	0	0		8.91	12.31	3.14	3.03	.75	.58
3 Camouflaged Words - NST01A	0	0		5.52	7.35	3.01	3.22	.74	.75
4 Circle Reasoning - CSS01C	++	0		4.26	6.33	2.72	2.60	.30d	.38d
5 Correlate Completion II - NSR01A	++	-c		10.66	23.23	9.17	10.04	.97	.92
6 Form Reasoning - NSI02B	++	-b		11.76	16.53	6.78	6.45	.78e	.78e
7 Letter Number - NSI03A	++	-c		6.06	9.51	3.89	3.38	.83	.73
8 Letter Series - NSR02B	0	0		9.28	13.38	3.85	4.16	.76	.72
9 Letter Triangle - CSS02A	0	0		6.61	8.87	2.64	2.70	.47	.50
10 Matched Verbal Relations - EMR02A	0	0		9.37	14.00	3.33	4.48	.55	.47
11 Necessary Facts - CMS04A	++	+		2.34	4.93	2.00	2.81	.47	.55
12 Number Rules - DSR02B	0	0		16.32	23.62	5.78	5.86	.68e	.71e
13 Numerical Operations - MSI01A	0	0		57.84	66.33	10.60	12.32	.82	.79
14 Picture Arrangement - NMS02A	0	0		9.82	11.08	2.28	2.35	.62d	.59d
15 Right Order Test - NSS01B	++	+		6.50	9.62	2.60	3.01	.33	.46
16 Seeing Trends II - CSR01A	++	0		5.66	9.92	2.41	3.36	.52	.68
17 Sentence Order - NMS03A	0	0		7.55	9.55	2.50	2.52	.91	.78
18 Ship Destination Test - Form A-2 (CMS)	++	-c		16.21	25.35	7.82	9.09	.40	.41
19 Sign Changes - NSI01A	-b	-b		29.59	36.66	7.93	4.81	.60c	.60c
20 Symbol Grouping - CSI01A	++	+		4.35	10.21	4.55	6.48	.80e	.61e
21 Word Changes - NSS02B	++	+		5.97	9.03	3.33	2.81	.80e	.89e
22 Word Linkage - EMR03A	++	-b		19.81	23.10	4.35	3.79	.80e	.81e
23 Word Patterns - CSI03B	0	-c		55.62	63.33	14.23	12.73	.62c	.65c
24 Word Relations - CSR02B	++	+		9.84	15.88	5.25	5.07	.59	.60
25 Word Transformation - NST02A	++	+		3.49	6.54	2.41	3.36	.83	.63
26 CTMM Language MA	-	0		160.71	185.06	19.08	17.40	.50d	.49d
27 CTMM Non-Language MA	-	0		156.50	175.64	22.11	22.34	.69d	.67d
28 Iowa Reading Comprehension - Test R	-	0		79.24	93.34	10.94	11.32	.68d	.71d
29 Iowa Arithmetic Concepts - Test A-1	-	+		79.23	92.92	10.64	10.54		
30 Iowa Arithmetic Problem Solving - Test A-2	-	+		79.22	92.31	9.42	9.95		
31 DAT Verbal Reasoning	+	+		11.93	20.59	5.81	8.45		
32 DAT Numerical Ability	++	+		9.61	19.95	5.60	6.48	.59d	.60d
33 DAT Abstract Reasoning	++	+		22.18	30.81	8.02	8.18		
34 DAT Clerical Speed and Accuracy	0	+		47.11	53.29	9.90	10.09		
35 SEX	-	+		0.44	0.50	0.50	0.50		
36 Mathematics Achievement Test	-	+		14.37		5.83			
37 Algebra Achievement Test	0	+		15.20	10.68	3.31	7.21	.47	.47
38 Abbreviations - ESI01A	0	+		11.58	16.39	2.48	3.74	.35	.35
39 Condensations - EST01A	0	+		11.29	12.85	3.01	2.34	.31	.31
40 Letter-Number Scales - ESS01A	0	0		4.51	11.63	10.02	3.24	.83e	.83e
41 Limited Words - DSI02B	-b	-b		11.35	3.78	7.63	5.08	.50	.50
42 Most Similar Sets - ESS02A	0	0		11.92	12.62	14.09	7.36	.82	.82
43 Number Combinations - DSS02B	+	0		11.92	17.16	7.11	14.93	.47	.47
44 Sign Changes II - ESR01B	+	0		11.27	12.47		3.13		

a The skewness of score distributions is described as follows: ++, strong positive skewness; +, slight positive skewness; 0, normal distribution; -, slight negative skewness; and --, strong negative skewness.

b Total scores dichotomized at the media for intercorrelations.

c Total scores C-scaled for intercorrelations.

d Communalities estimates entered as reliability estimates for one-part tests.

e Spearman-Brown reliability estimates on all dichotomized variables are based on tetrachoric inter-part correlations. Estimates for all remaining variables are based upon Pearson-r coefficients.

Table 9
General Mathematics Correlation Matrix^a (N = 217)^a

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44
1	14	19	12	28	19	14	34	21	14	28	34	42	23	31	25	21	28	34	16	21	17	29	17	09	29	24	21	29	28	18	42	31	21	-03	40	13	16	11	19	19	36	23	
2	14	19	27	19	10	05	27	30	40	28	19	13	29	32	36	28	27	15	21	21	30	22	32	26	41	16	32	22	19	41	31	33	11	-00	29	28	12	-03	29	09	27	15	
3	19	19	27	21	29	38	29	31	24	30	19	24	27	16	27	26	11	23	09	26	29	30	28	24	33	20	21	18	13	28	27	22	25	-04	23	11	16	05	19	16	37	26	
4	12	27	21	29	24	14	25	21	26	23	13	06	09	08	20	20	11	03	15	25	24	11	26	27	23	17	22	14	10	27	20	23	13	-05	18	14	12	-12	16	-05	19	03	
5	28	19	29	29	31	39	50	35	40	38	39	29	27	26	49	37	34	20	25	51	34	33	51	36	40	31	31	36	11	40	44	49	22	-05	42	36	38	-01	35	20	37	21	
6	19	10	38	24	31	31	35	15	27	28	28	08	21	34	24	24	31	25	31	24	33	39	39	21	18	20	24	23	26	33	21	21	-10	28	10	23	06	17	19	10	19		
7	14	05	29	14	39	31	34	19	06	25	33	09	06	08	18	17	19	-04	16	36	08	07	19	25	16	22	13	16	12	05	25	20	05	01	12	02	17	-03	23	01	26	14	
8	34	27	33	25	50	35	34	36	25	26	23	26	20	24	33	23	26	19	11	46	14	24	39	32	28	19	23	22	24	24	32	38	38	-20	32	16	27	-07	28	10	32	21	
9	21	30	19	21	35	13	19	36	24	28	19	25	19	19	38	10	30	19	09	21	22	15	33	22	32	23	24	15	15	24	31	28	18	-03	23	12	19	-02	23	09	20	17	
10	14	40	24	26	40	15	04	25	24	24	38	18	10	31	19	31	24	27	33	13	28	42	22	13	18	43	30	37	15	18	43	31	34	10	05	29	10	19	14	20	08	26	
11	28	28	20	23	38	27	25	26	28	38	26	21	26	30	37	24	33	35	40	28	28	24	43	37	33	19	25	14	10	34	40	35	26	01	35	19	10	08	10	17	24	19	
12	34	19	19	13	39	28	33	23	15	18	26	15	16	17	22	34	13	21	22	26	26	14	29	28	22	32	31	14	27	25	34	20	35	17	45	-20	39	17	22	07	10	12	20
13	42	13	24	06	29	28	09	26	25	10	21	34	14	29	29	27	08	56	24	28	20	24	35	34	32	23	19	37	26	28	35	21	23	06	43	06	20	10	16	10	36	21	
14	23	29	27	09	27	08	04	20	19	31	26	19	14	15	16	17	22	34	13	21	22	26	26	14	29	26	35	25	16	28	31	39	16	-03	30	00	11	15	20	20	37	26	
15	31	32	16	08	26	21	08	24	19	19	30	24	29	15	32	18	21	21	25	21	21	29	24	31	22	19	25	15	19	31	26	12	01	27	02	10	04	19	13	34	25		
16	25	36	27	20	49	34	18	33	38	31	37	27	29	16	32	20	16	24	31	32	24	33	36	40	17	33	28	19	43	36	37	18	03	36	24	25	07	34	15	37	28		
17	21	28	26	20	37	24	17	23	10	24	24	17	27	17	18	14	11	28	23	23	35	24	34	44	13	44	24	18	42	35	18	19	-10	29	32	22	01	09	09	20	19		
18	28	27	11	11	34	24	19	26	30	27	33	17	08	22	21	20	11	26	17	24	25	32	18	26	31	23	28	14	20	36	33	44	58	27	42	23	22	09	40	19	31		
19	34	15	23	03	20	31	-04	19	19	23	35	35	56	34	21	16	20	11	06	13	19	30	29	30	29	19	26	24	21	25	32	33	01	20	27	17	16	13	17	08	29	11	
20	16	21	09	15	25	25	16	11	09	13	40	24	24	13	25	24	23	26	06	27	21	23	25	23	23	24	16	06	16	21	24	19	15	-06	24	02	13	02	17	10	18	10	
21	21	21	26	25	51	31	36	46	21	28	20	28	19	21	21	33	23	19	13	27	13	21	33	35	16	31	18	11	05	19	20	48	25	-08	20	11	00	-01	15	14	21	15	
22	17	30	27	09	26	34	24	08	14	22	42	28	20	19	22	21	32	35	24	19	21	13	25	18	22	31	29	22	23	26	31	25	26	-11	22	26	20	07	30	04	30	28	
23	29	22	30	11	33	33	07	24	15	22	24	28	26	21	24	24	25	30	23	31	31	18	28	28	30	26	23	14	37	36	27	24	-06	32	22	31	02	33	22	32	21		
24	17	32	28	26	51	39	19	39	33	31	43	25	22	26	29	33	34	32	29	25	33	31	18	28	30	26	23	14	37	36	27	24	-06	32	22	31	02	33	22	32	21		
25	09	26	24	27	36	39	25	32	22	18	37	34	32	14	24	36	23	18	30	30	22	28	30	16	19	20	20	29	31	27	27	-01	31	24	18	-05	28	22	27	27			
26	29	41	33	25	40	21	16	28	32	43	33	32	31	29	31	40	44	26	29	23	16	49	31	28	30	33	63	34	36	57	49	36	18	-10	42	18	27	09	33	14	45	34	
27	24	16	20	17	31	18	22	19	23	30	19	23	14	26	22	17	13	31	19	24	31	29	30	16	33	25	33	33	30	40	47	07	18	30	04	20	11	26	11	30	24		
28	21	32	21	22	31	20	13	23	24	37	25	19	27	35	19	33	44	23	26	14	10	46	22	26	19	63	25	47	44	57	39	24	15	-04	32	09	22	02	26	03	34	33	
29	29	22	18	14	26	26	16	22	15	15	14	17	27	25	25	28	24	28	24	06	11	23	23	23	20	34	33	47	58	34	50	31	06	18	46	11	16	08	30	13	38	31	
30	28	15	13	10	11	23	12	24	15	18	10	26	34	16	15	19	18	21	24	16	05	16	26	14	20	36	33	44	58	27	42	23	22	09	40	19	31	17	36	29			
31	18	41	28	27	40	26	05	24	24	43	34	28	20	28	19	43	42	25	29	21	19	53	31	27	29	57	57	34	27	45	36	12	02	35	25	10	16	29	18	32	33		
32	42	31	27	20	44	33	25	32	31	51	40	35	35	33	31	36	35	32	31	24	20	25	32	36	31	49	40	39	50	42	45	44	16	05	63	18	22	08	30	26	08	35	
33	31	33	22	23	49	21	20	38	28	34	35	21	17	39	26	37	18	33	16	19	48	26	27	27	36	47	24	31	23	36	44	11	-02	35	12	21	05	32	14	37	22		
34	21	11	25	13	22	21	05	38	18	18	26	23	45	16	12	18	19	01	41	15	25	17	22	24	27	18	07	15	06	22	12	16	11	-36	24	25	21	02	10	15	20	19	
35	-03	-09	-04	-05	-05	-10	01	-20	-03	05	01	06	-20	-03	01	03	-10	20	-17	-06	-08	-11	-08	-06	-01	-10	18	-04	18	09	02	05	-02	-36	04	-21	-12	13	-01	05	-01	-03	
36	40	29	23	18	42	28	12	32	23	29	35	43	39	30	27	36	29	27	43	24	20	22	27	32	31	42	30	32	46	40	35	63	35	26	04	27	18	13	25	31	41	44	
37	13	28	11	14	36	10	02	16	12	18	19	06	17	08	02	24	32	17	15	02	11	26	30	22	24	18	04	09	11	03	25	18	-20	-21	17	17	04	16	16	26	05		
38	16	12	16	12	38	23	17	27	19	19	10	20	22	11	18	25	22	16	17	13	08	20	20	31	18	27	28	22	16	10	23	22	21	-12	18	17	-03	20	17	13	15		
39	11	-03	05	-12	-01	06	-03	-07	-02	14	08	10	07	15	04	07	01	13	16	02	01	07	13	02	-05	09	11	02	08	16	05	08	15	02	13	04	-03	-01	12	08	24		
40	15	25	19	16	35	17	21	28	23	20	10	16	10	20	19	34	09	17	-04	17	15	30	23																				

essentially Pearson-r coefficients. Those correlations between and with binary scores have been adjusted in value to be comparable with Pearson r's. The scaled-score intercorrelations were not adjusted, since the correction for eleven-category coarse grouping is negligible. The correlation matrices as presented are therefore considered unbiased.

It should be noted that each correlation matrix is segregated into two sections; those variables numbered 1 through 37, and those numbered 38 through 44. This segregation was made because there was not complete overlap of examinees in the two major testing sessions. Some attrition resulted in the second testing administration. Since the N does not vary much in either case, however, any bias due to loss of degrees of freedom is minimal. Since the factor analyses were based upon the first test administration only, this segregation will not affect the factor structures.

The Factor Matrices. — The unrotated, principal-components factor matrices are given in Tables 11 and 12. Extraction was discontinued after the fourteenth factor due to small variation of factor loadings and the small size of the eigenvalue with less than 5 per cent of the total variance remaining in the residual matrix.

The rotated factor matrices are presented in Tables 13 and 14. For both groups, one of the 14 factors was residualized, and 13 factors were interpretable in terms of the variables defining them.

Interpretation of Factors

In the following paragraphs, the rotated factors will be treated in turn, with matched results from the two analyses. The tests significantly loaded (.30 or higher) on each factor are listed for either course sample. If there are also significant loadings on other factors for a test, those loadings are mentioned in parentheses. Interpretations pay most attention to tests without secondary loadings on other factors, but, owing to the fact that, in most cases, only two tests were employed to define each factor, many tests do not appear univocal nor do factors appear well defined.

CSR - Cognition of symbolic relations

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
24. Word Relations (CSR)	.02 (NSR .38)	.48 (NSR .38)
16. Seeing Trends II (CSR)	.39	.47
9. Letter Triangle (CSS)	.33	.25 (NSR .52)

The two tests hypothesized to define CSR, Word Relations and Seeing Trends II, appear to have functioned well in the algebra group, but the former did not help define CSR for the general-mathematics group. There does seem to be some confusion between the factors CSR and NSR. Tests of these two factors have never been analyzed together previously and it appears that the measures of the two factors are not as independent of each other as has been thought.

CSS - Cognition of symbolic systems

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
4. Circle Reasoning (CSS)	.42	.53

Although two tests were hypothesized to define factor CSS, one of the tests, Letter Triangle, consistently shares its variance with other factors. In a previous study (Guilford, et al, 1961), Letter Triangle was relatively univocal, as was Circle Reasoning. A possible reason for Letter Triangle's loading on relations factors (CSR and NSR) is that some of the answers can be obtained from only one or two relations; thus, a system might not necessarily need to be recognized. Circle Reasoning seemingly performed as well as it ever has, in this study. It seems that to perform well on this test, the system must be grasped; seeing a relation would not necessarily yield a correct response.

CSI - Cognition of symbolic implications

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
6. Form Reasoning (NSI)	.29 (NST .39)	.50
23. Word Patterns (CSI)	.40	.36

Only one hypothesized CSI test functioned as expected. Word Patterns previously had a univocal loading on this factor. The second CSI test, Symbol Grouping, which previously had secondary loadings on systems factors (CSS and NSS), failed to load significantly on this factor. Form Reasoning was intended to be used as a measure of symbol substitution. For this study the format was altered so that the task of substitution was more orderly and easy. This simplification may have resulted in the task becoming one of quickly cognizing the implication. If that is the case, this result would be an interesting example of how easily a small change alters a test's factor composition.

CMU - Cognition of semantic units (verbal comprehension)

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
11. Necessary Facts (CMS)	.05 (CMS .45)	.66
26. CTMM Language MA (CMU)	.41 (NMS .42)	.50 (EMR .45)
28. Iowa Reading Comprehension (CMU)	.48 (NMS .50)	.48 (NMS .30, EMR .45)
22. Word Linkage (EMR)	.45	.20 (EMR .58)

Table 11
General-Mathematics Factor Matrix - Principal Components^a

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	h^2
1	47	-04	33	-25	-16	-21	-05	-01	15	-05	-01	08	-13	15	53
2	51	-16	-29	-16	04	07	17	-08	01	-08	-09	-11	12	02	48
3	48	08	04	21	-12	16	-20	11	07	-14	-20	02	-10	-07	45
4	37	09	-25	13	02	06	06	03	00	-06	-05	05	03	23	30
5	72	23	-17	-06	-12	-02	07	-14	-03	05	10	-19	-12	-06	71
6	52	27	16	27	15	08	-11	21	16	-00	08	-05	08	03	56
7	37	47	-05	14	-12	-23	-20	05	-21	02	-02	03	-03	-06	54
8	57	33	-02	-07	-31	09	07	08	12	02	00	07	07	08	58
9	46	03	-09	-22	-17	05	20	24	-12	-02	04	09	-02	-11	44
10	52	-20	-27	-08	-01	18	-06	-10	-04	-09	01	01	-08	07	46
11	60	04	00	-17	31	09	-12	04	-16	07	-18	14	-10	-01	61
12	50	11	25	02	03	-16	-04	-15	-24	-14	10	-12	-03	08	49
13	50	-09	54	07	-04	-03	23	-01	01	03	03	09	-04	-06	62
14	43	-21	-01	-17	-13	19	-26	-16	-04	-00	-05	00	10	-09	42
15	45	-03	08	-15	14	-08	16	-01	11	-04	-09	00	02	04	31
16	60	04	-10	02	03	-06	27	05	04	-03	-10	-05	-06	-15	49
17	51	-16	-04	27	-01	-09	02	-10	11	25	-07	-07	-04	01	47
18	45	03	-10	-33	11	-09	-17	15	07	-01	23	03	06	-04	46
19	48	-25	54	01	-01	31	-01	-03	-13	07	07	-00	01	01	71
20	40	09	-00	-05	41	-19	-03	-14	10	06	-01	10	03	-06	42
21	51	43	-10	02	-10	08	03	-33	08	01	02	15	04	-01	62
22	53	-29	-23	21	13	01	-02	05	02	-13	17	04	-17	02	56
23	47	-07	15	-01	03	04	-14	-04	24	-17	02	-10	05	-11	38
24	60	12	-11	-06	11	21	01	10	01	27	07	-10	-06	06	55
25	53	17	08	19	19	09	15	02	-16	-13	02	02	17	00	49
26	67	-37	-13	13	-08	-22	02	05	-05	-07	-04	02	03	01	69
28	57	-41	-16	20	-17	-12	-06	-01	-02	15	10	14	12	-04	67
32	65	-10	11	-13	-05	-18	-12	12	-06	08	-14	-15	11	07	60

Table 12
Algebra Factor Matrix - Principal Components^a

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	h^2
1	51	-34	07	-12	32	06	14	09	05	-11	-14	-00	12	14	60
2	48	28	-08	-23	-08	-04	-10	11	-11	-18	-08	-16	03	05	47
3	39	-12	20	10	-03	17	06	07	-04	-19	22	-05	08	-14	37
4	39	09	08	05	-02	-06	-31	-03	-15	16	-05	-21	13	-03	38
5	72	-11	-25	-00	-11	-05	02	15	-08	05	05	01	08	11	66
6	43	-29	-03	31	10	14	-34	08	11	-07	-01	02	-00	01	53
7	48	-28	-23	03	-10	-06	10	18	-12	-06	13	02	-02	10	46
8	67	-22	-18	02	-07	-12	-08	-14	18	05	02	15	12	00	64
9	44	-04	-42	-12	-02	-05	14	-16	02	14	-05	10	04	-06	48
10	61	40	-10	10	-06	03	05	-13	-12	-18	-12	00	08	-06	65
11	63	24	17	-18	01	-05	-29	25	-00	13	07	15	-01	08	72
12	67	-15	12	-21	19	-03	09	08	12	-05	07	-08	02	-10	62
13	33	-43	38	-27	-04	10	-01	-17	-13	-03	-10	05	-08	04	59
14	45	10	06	31	-07	-18	-04	-09	11	06	-02	-01	-03	02	37
15	55	-22	03	-10	03	13	00	16	13	15	-23	-03	-05	-17	54
16	54	01	-04	-08	-23	13	-19	-09	-16	-06	-05	11	-17	-07	49
17	46	06	22	30	-23	-14	14	12	12	01	-15	-13	-09	06	52
18	62	07	-19	05	33	-12	-11	10	-03	-07	00	07	-12	-10	61
19	31	-38	43	12	01	-32	05	-12	-20	10	02	04	05	-02	62
20	50	-00	-15	09	31	-33	03	-13	-07	-07	07	-09	-12	-02	54
21	64	-03	-16	33	07	32	25	05	-17	14	-07	-01	-05	04	77
22	54	29	10	09	18	17	-08	-23	-03	-03	-08	08	13	05	55
23	39	-15	-01	10	-05	18	-13	-22	23	-08	11	-10	-06	09	38
24	63	-20	-13	-12	-40	-13	02	-01	07	-16	-04	-01	04	-05	68
25	58	06	13	10	-10	15	08	04	-04	12	24	00	04	-09	49
26	60	46	18	-12	-01	-04	16	02	12	06	01	02	08	-04	67
28	65	36	29	-03	-00	-01	15	-01	07	-07	05	13	-13	08	71
32	58	04	-11	-32	04	09	03	-18	02	18	12	-20	-10	07	60

^a Decimal points omitted.

Table 13

General-Mathematics Factor Matrix - Graphic Solution^a

	CSR	CSS	CSI	CMU	CMS	MSI	DSR	NSR	NSS	NST	NSI	NMS	EMR	RES	h^2
1	05	04	18	16	19	24	48	06	12	-06	26	01	22	11	54
2	28	27	28	07	07	06	-11	10	07	06	03	21	37	-12	47
3	08	08	12	07	10	-11	26	-02	06	50	12	20	09	07	44
4	18	42	04	05	08	-11	03	11	10	14	09	14	05	-04	31
5	27	12	27	01	20	05	21	46	29	24	05	15	20	-19	70
6	-06	20	29	19	20	01	14	12	17	39	23	13	-25	00	55
7	-01	18	-08	-03	25	15	29	29	19	39	-19	-03	-11	-03	53
8	25	21	18	-05	18	08	17	33	27	28	22	04	08	25	59
9	33	09	05	05	16	25	-01	29	-09	21	21	04	20	05	43
10	20	20	09	20	11	15	01	12	08	16	12	19	43	-14	46
11	-08	22	07	05	45	17	05	01	05	28	23	20	29	-22	60
12	-04	14	12	16	-01	29	33	13	27	19	09	-02	10	-29	48
13	13	-12	15	17	02	40	28	-04	22	10	42	21	-04	-07	61
14	-05	-03	11	19	09	00	02	05	18	16	06	13	49	07	39
15	16	17	28	06	20	22	10	-02	07	-00	18	12	13	-09	31
16	39	13	26	02	20	19	10	12	06	22	10	25	07	-15	49
17	11	07	16	15	12	00	18	11	19	06	02	55	01	-09	47
18	02	13	24	29	39	10	00	29	-02	07	06	-11	21	00	47
19	-16	-15	10	25	-11	24	11	-02	20	20	54	26	20	-08	69
20	00	16	22	14	44	14	03	-07	20	-02	-01	09	-01	-24	47
21	22	15	09	-10	29	-02	10	18	57	24	06	00	08	-00	61
22	28	17	09	45	12	-18	09	08	-03	15	10	26	13	-27	57
23	06	-02	40	26	11	01	17	-04	15	22	11	08	15	02	39
24	02	17	22	03	28	-02	-04	38	10	20	29	28	12	-11	54
25	11	29	14	13	05	23	-03	03	26	36	20	09	-08	-22	49
26	29	24	12	41	05	18	22	08	01	13	-04	42	26	-07	68
28	22	09	-02	48	07	09	07	17	14	07	-03	50	21	07	66
32	-05	22	29	17	14	33	27	21	03	16	06	29	26	03	59

Table 14

Algebra Factor Matrix - Graphic Solution^a

	CSR	CSS	CSI	CMU	CMS	MSI	DSR	NSR	NSS	NST	NSI	NMS	EMR	RES	h^2
1	-08	-05	21	03	15	20	52	17	30	02	29	11	07	07	61
2	24	28	-07	29	22	09	27	06	03	01	-06	07	28	20	49
3	03	03	14	05	14	09	12	11	06	48	14	12	05	15	37
4	05	53	12	13	10	10	04	05	08	06	05	15	13	-07	38
5	24	25	11	27	11	08	19	49	25	04	20	19	11	21	70
6	12	18	50	-04	-01	-16	17	15	23	15	04	23	03	-02	51
7	24	08	12	10	10	05	15	42	18	06	20	14	-10	29	50
8	25	11	24	11	-05	10	22	53	10	05	26	21	22	-10	67
9	25	04	-05	07	04	13	07	52	25	-11	09	-05	22	-05	50
10	16	20	-02	18	29	00	04	20	12	12	05	12	63	19	70
11	18	28	24	66	15	09	16	00	06	07	18	13	15	-04	74
12	13	02	14	22	27	29	44	23	18	23	17	20	08	-08	66
13	18	02	22	-08	-07	48	24	-12	05	14	41	01	-00	05	59
14	06	14	14	11	11	01	-09	21	-02	04	13	43	26	-04	39
15	26	08	09	13	03	14	29	07	47	15	21	20	05	-15	56
16	47	21	21	15	04	16	01	09	13	12	13	02	27	14	51
17	08	08	-03	12	07	04	-02	03	13	11	19	63	18	14	55
18	20	09	28	22	48	-05	18	28	22	-03	07	10	19	-07	62
19	-05	14	13	-10	14	29	-01	06	-10	10	58	27	-12	-05	60
20	09	09	20	-00	51	13	11	37	03	-12	08	21	14	-04	57
21	00	08	18	09	17	09	-08	32	62	17	15	18	29	28	82
22	-10	17	25	20	17	12	07	10	14	10	09	04	58	02	58
23	12	04	36	-05	-13	18	13	22	07	17	-10	23	19	04	40
24	48	16	-04	08	-06	13	27	38	02	14	22	29	17	16	71
25	06	14	13	29	10	20	-05	25	18	40	13	18	17	08	52
26	01	07	-12	50	22	20	11	11	07	17	07	24	45	-03	68
28	08	-07	11	48	26	25	05	03	03	17	16	30	45	11	75
32	18	16	09	24	11	52	18	30	20	03	-12	05	19	-01	62

^a Decimal points omitted.

The verbal-comprehension factor was measured consistently by the two published subtests hypothesized to measure CMU. In each group, however, another test shared variance on this factor. Without any apparent reason, Necessary Facts, previously a CMS test, appears to become a CMU measure for algebra students. This kind of result might have been expected more in the general-mathematics group, where CMU ability is generally lower and perhaps more variable. Word Linkage, a test developed for the factor EMR, had never before been factor analysed. It may actually contain considerable CMU variance.

CMS - Cognition of semantic systems (general reasoning)

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
18. Ship Destination Test (CMS)	.39	.48
20. Symbol Grouping (CSI)	.44	.51
11. Necessary Facts (CMS)	.45	.15 (CMU .66)

There is considerable evidence that Ship Destination, a test of factor CMS, is a measure of the ability to structure problems meaningfully in preparation for solving them. On this basis, Ship Destination and Necessary Facts were hypothesized to define CMS, which they both did in the general-mathematics group. The test Symbol Grouping, developed to measure CSI, however, is loaded on this CMS factor for both samples. Each item in this test involves a sequence of three kinds of symbols, which makes it a system. It may be that these students tended to attack the items as semantic-order problems, which would account for the CMS variances.

MSI - Memory for symbolic implications (numerical facility)

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
32. DAT Numerical Ability (MSI)	.33	.52 (NSR .30)
13. Numerical Operations (MSI)	.40 (NSI .42)	.48 (NSI .41)

The two tests hypothesized to define the numerical-facility factor functioned partly as expected. Numerical Operations had high secondary loadings on the NSI factor, a factor parallel to MSI, in both groups. Apparently the distinction between the factors MSI and NSI, two factors never separated before in the same study, is less clear than was expected. This result, if it holds up in future replications, has two interesting implications. On the one hand, it accounts for the earlier placement of the long-recognized numerical-facility factor in cell NSI. On the other hand, it suggests that a number-operations test may not be univocal as formerly supposed.

DSR - Divergent production of symbolic relations

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
1. Alternate Additions (DSR)	.48	.52 (NSS .30)
12. Number Rules (DSR)	.33	.44

Number Rules was found to be loaded on factor DSR in its only one previous analysis (Gershon, et al, 1963). In that study, Alternate Additions, although designed for DSR, was found loaded on factor MSI in two analyses. This present solution, then, can be said to have succeeded in finding factor DSR more clearly than before.

NSR - Convergent production of symbolic relations (symbolic correlates)

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
8. Letter Series (NSR)	.33	.53
9. Letter Triangle (CSS)	.29 (CSR .33)	.52
5. Correlate Completion II (NSR)	.46	.49
7. Letter-Number (NSI)	.29 (NST .39)	.42
24. Word Relations (CSR)	.38	.38 (CSR .48)
20. Symbol Grouping (CSI)	-.07 (CMS .44)	.37 (CMS .51)
21. Word Changes (NSS)	.18 (NSS .57)	.32 (NSS .62)
32. DAT Numerical Ability (MSI)	.21 (MSI .33)	.30 (MSI .52)

Factor NSR functioned in both analyses as a somewhat general-symbolic factor, since many of the test loadings could not be rotated off of it and distributed over other factors. It should be noted that Letter Series and Correlate Completion II have never defined a factor by themselves, but have been loaded on complex factors with tests of CSR and CSS. Although the test Letter-Number was written as a test of NSI, after inspecting student responses, the investigators felt it to be a better measure of NSR, because the items were easier than had been expected. Apparently, seeing a relation is more important than seeing an extension or implication. Although the NSR tests load on this factor, there is obvious need for the clarification and refinement of the measures.

NSS - Convergent production of symbolic systems

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
21. Word Changes (NSS)	.57	.62
15. Right Order Test (NSS)	.07	.47
1. Alternate Additions (DSR)	.12 (DSR .48)	.30 (DSR .52)

Word Changes and Right Order Test (formerly named Operations Sequence) had defined the factor NSS in a previous study. In the algebra analysis they functioned in the same manner to define this symbol-ordering ability. The factor emerged as a singlet in the general-mathematics group, however. The absence of NSS variance in the

Right Order Test for the general-mathematics group may be due to the difficulty of that test. The communality of the test was only .31 in that group but .56 in the algebra group.

NST - Convergent production of symbolic transformations

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
3. Camouflaged Words (NST)	.50	.48
25. Word Transformation (NST)	.36	.40
6. Form Reasoning (NSI)	.39	.15 (CSI .50)
7. Letter-Number (NSI)	.39	.06 (NSR .42)

The two tests hypothesized to define factor NST functioned as expected in a thoroughly univocal manner. This same factor emerged in a previous study (Guilford, et al, 1961). It seems clear that this factor is concerned with the ability to segregate a group of consecutive symbols to form a new unit. About all that can be said about the presence of Form Reasoning and Letter-Number in the list for the general-mathematics group is to point out that they were designed for the neighboring factor NSI, to be discussed next. In the algebra group those tests did not function as NSI tests either, but went with factors CSI and NSR, respectively.

NSI - Convergent production of symbolic implications

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
19. Sign Changes (NSI)	.54	.58
13. Numerical Operations (MSI)	.42 (MSI .40)	.41 (MSI .48)

If factor NSI had emerged more clearly from a factor matrix containing also the factor MSI, it would have been possible to say with more assurance that these analyses had accomplished the first separation of these two factors. Form Reasoning, meant to define NSI, had been altered, and Letter-Number seemed a more reasonable measure of NSR. From this solution, it appears that Sign Changes is a univocal measure of NSI, but that then the Numerical Operations test, which has always been regarded as a univocal measure of factor MSI, is a test of complexity two. This point will have to be ironed out in an analysis with more tests of both factors in the battery.

NMS - Convergent production of semantic systems

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
17. Sentence Order (NMS)	.55	.63
14. Picture Arrangement (NMS)	.13 (EMR .49)	.43
28. Iowa Reading Comprehension (CMU)	.50 (CMU .48)	.30 (EMR .45)
26. CTMM Language MA (CMU)	.42 (CMU .41)	.24 (CMU .50, EMR .45)

Factor NMS was defined by the tests Sentence Order and Picture Arrangement. These tests measure the ability to order conceptual information and have defined this factor rather consistently in factor-analytic studies. Once again, this ordering factor was distinctly separable from its parallel symbolic factor, NSS. One of the surprises here is the fact that the two marker tests for CMU showed substantial NMS variance. This is the first time, to our knowledge, that academic-aptitude tests have shown relationship to the ability to produce correct order.

EMR - Evaluation of semantic relations (logical evaluation)

<u>Tests</u>	<u>General Mathematics</u>	<u>Algebra</u>
10. Matched Verbal Relations (EMR)	.43	.63
22. Word Linkage (EMR)	.13 (CMU .45)	.58
2. Best Trend Name (EMR)	.37	.28
14. Picture Arrangement (NMS)	.49	.26 (NMS .43)
26. CTMM Language MA (CMU)	.26 (NMS .42, CMU .41)	.45 (CMU .50)
28. Iowa Reading Comprehension (CMU)	.21 (NMS .50, CMU .48)	.45 (CMU .48, NMS .30)

Factor EMR was concurrently under investigation in another study while the battery for the algebra study was being formed. The concepts of what kinds of tests measure "logical evaluation," as the EMR factor was originally named, had undergone extensive changes. Previously, the test Logical Reasoning, a multiple-choice syllogism test, had been regarded as typical. Since that test was not in the battery of the algebra study, the obtained EMR factor may be an entirely new one. It appears that the two marker tests for CMU share some common-factor variance with factor EMR as well as with NMS, as just previously noted, apparently shedding more light on the additional common-factor variances in these primarily CMU tests. Curiously, the expected secondary variance in factor CMS did not show up. One might not expect it to do so in a vocabulary test, but it commonly does in a reading-comprehension test.

Congruence of Factors

Two incidental problems in this investigation were concerned with the degree of similarity of factor structure between general-mathematics and algebra students, on the one hand, and between boys and girls, on the other. Application of Tucker's coefficient of congruence to the two pairs of rotated factor matrices gives a basis for determining whether pairs of factors, interpretatively matched, have been properly matched in terms of objective evidence.

In the comparison of matched factors in the two mathematics groups, the Tucker coefficients were found to range from .56 to .85, whereas in unmatched pairs they ranged from .10 to .76. In four cases out of 13, for matched factors, the coefficient was a little higher for some other pairing. But no re-pairings would give an over-all better matching than the meaningful matchings yielded. It has been found on the basis of experience that no rigid minimum ϕ_r coefficient can be adopted as the criterion for an appropriate matching of two factors. Typical coefficients are in the range of .70 to .85.

In the comparison of factor structures for boys and girls, the varimax factors that appeared by inspection to be most similar were paired. The Tucker coefficients for these pairs ranged from .57 to .93 (ignoring a residualized factor), with the range for unmatched pairs being .10 to .80. It is apparent that the matching is about equally good in the two comparisons, but in both cases the relatively low Tucker coefficients reflect the fact that, although one can say that the factors are properly matched, there are some noticeable differences in the way that tests function in two populations. In the absence of criteria for rejecting similarity of over-all structures, we see no reason for rejecting the analyses with the two sexes included. Other reassuring evidence is that the factor tests correlated with sex membership only as much as -.20 in the general-mathematics group and as much as -.23 in the algebra group.

Multiple-regression Analysis

In order to answer the questions posed in the section on specific steps, 56 regression equations were computed for predicting the several criteria from the combinations of tests listed in Table 6. Table 15 presents all the pertinent data from these multiple-regression equations. The complex format of this table requires some explanation.

In the left-hand column all the predictors used in any aspect of this study are listed; first the 13 factor scores, then the 2 CTMM subscores, the 3 Iowa subscores, the 4 DAT subscores, and the 7 test scores from Booklets G and H. Below all that are listed the criteria; the traditionally scored general-mathematics-achievement test and its construct scoring; the traditionally scored algebra-achievement test and its construct scoring; and last, the course grade.

The table is divided into four major, vertical sections, one section for each homogeneous group of students. Within each section is a column for test-score means and one for test-score standard deviations.⁸ To the right of these two columns is a series of 14 columns of beta weights. Each of the 14 columns is headed by a number. This number refers to the equation number listed in Table 6. Therefore, if from Table 6 we want to inspect the prediction of a traditionally scored achievement test as predicted by the DAT subscores, we would find that the appropriate equation for the Basic-Mathematics and Non-college-Algebra groups is equation No. 7. Looking down the column numbered 7 in the section labeled "Basic Mathematics," we find four beta weights, one for each DAT subtest, a coefficient of multiple correlation for the traditionally scored general-mathematics-achievement test, the significance level of that coefficient, and a shrunken estimate of the coefficient. Thus, for any of the 56 equations, we need only choose the correct group and appropriate equation number from Table 6 to find all the pertinent statistical data for that prediction in Table 15.

Relative Importance of the Factors.—In this study, we look for evidence concerning the relative importance of the factors in the beta coefficients of the multiple-regression equations, obtained in connection with the prediction of achievement. The clarity of the conclusions on these points should be enhanced from the fact that the beta coefficients, like coefficients of partial correlation, to which they are closely related, indicate relationships freed from the concomitant influences of other predictors. One disadvantage, in the present instance, is that we are dealing with predictors in the form of test-score composites that do not represent their factors exactly. Also, one reservation to be remembered is that the betas are obtained in the context of certain combinations of predictors.

Table 16 has been compiled for convenience in comparing the beta weights that may have some claim to psychological significance. The beta weights were used rather than the beta-r products because they give estimates of the direct contributions of the variables, whereas the beta-r products include indirect contributions. Also, there were some negative betas in the equations, under which condition the beta-r products are of questionable meaning.

The computer program yielded t-test data for the various b coefficients, which may also be applied to testing the statistical significance of the beta coefficients. The starred betas in Table 16 indicate that very few of those coefficients achieved significance at the .05 level or better. There is some question about the meaningfulness of such t tests when the numbers of variables are relatively large, as in this instance. The larger the number of predictors, the less likely that many reach statistical significance. One can imagine the extreme case in which no beta is large enough, yet the multiple R is highly significant. All the R's associated with the regression equations involved in Table 16 were significant beyond the .01 level. At any rate, we shall see what information may be extracted from the larger betas, many of which do not achieve the .05 level of significance.

In Table 16 are listed arbitrarily the betas of .15 or greater in one or more of the three equations in each population, with the belief that the coefficients of this size merit some claim to attention. The rank orders of the betas for each population are determined primarily by those from the longest equation, which includes the 13 factor-score predictors plus the 7 factor tests. The shorter equations are for the 13 factor scores only and for the 7 factor tests only. Estimated, unbiased multiple R's are given at the bottom of the table as supplementary information.

⁸ Factor scores are composites of standard scores from factor tests.

12

^b Decimal points omitted from beta weights and correlation coefficients.

Table 16

Factor Predictors with the Larger Beta Weights in 12 Equations,
with Multiple-Correlation Coefficients

Group															
Basic Mathematics				Non-college Algebra			Regular Algebra			Accelerated Algebra					
MSI	.40 [*]	.42 ^{**}	--- ^a	ESR	---	.33 ^{**}	.50 ^{**}	MSI	.21	.25 [*]	---	DSR	.23 [*]	.20	---
NMS	.17	.30 [*]	---	MSI	.34 ^{**}	.22	---	NSI	.16	.19	---	NSR	.18	.18	---
DSI	---	.23	.12	EMR	.19	.22 [*]	---	ESS ₂	---	.19	.12	CMS	.17	.17	---
NST	-.08	-.19	---	ESS ₁ ^b	---	-.20	-.21 [*]	NSS	.16	.18	---	NSS	.14	.15	---
DSS	---	.06	.27 [*]	CMU	-.04	-.17	---	EMR	.17	.18	---	CSR	.15	.11	---
ESR	---	.12	.25 [*]	EST	---	.16	.09	DSR	.20	.08	---	DSS	---	.05	.33 ^{**}
				DSI	---	.13	.15	ESS ₁	---	.08	.20 [*]	EST	---	.08	.22 [*]
				NSR	.15	.06	---					ESI	---	.06	.18
												ESS ₂	---	.03	.18
												DSI	---	.08	.15
cR	.46	.48	.42		.45	.54	.56		.39	.38	.27		.75	.74	.51

^a In each case, the first of the three betas comes from the equation including the 13 factor-score predictors, the second from the equation with the 7 factor tests added, and the third from the 7 factor-test predictors alone. A blank means that the variable was not included in the equation.

^b There were two factor tests and hence two predictors for factor ESS.

* Significant at the .05 level; ** significant at the .01 level. All R coefficients were significant at the .01 level.

Inspection of Table 16 presents a few general impressions. First, there is somewhat of a trend for an increasing number of factors to be represented as we go from the basic-mathematics group to the accelerated-algebra group. It would be natural to expect greater factorial complexity as we go from mathematically less capable groups to more capable groups and from ordinary arithmetic to intermediate algebra.

A second general impression is that factors that have been known to dominate traditional academic-aptitude tests are conspicuous by their absence in the lists. The two dominant factors in such tests, CMU and CMS, appear each in only one group. Furthermore, CMU appears with negative sign. The latter result may mean that CMU is not actually involved in the criterion but it is involved in some of the semantic tests, such as for factor EMR, and the negative weight for the CMU score gives that score the status of a genuine suppressor variable. None of the remaining factors listed in Table 16 are known to carry much weight, if any, in traditional tests, particularly the symbolic tests, which dominate the picture in Table 16.

A third general impression is the apparently promising status of three symbolic-evaluation abilities (ESR, ESS, and EST). It should be remembered that the tests for these factors have not yet been factor analyzed. There is a natural question concerning the two tests for factor ESS, since one of them achieved negative weights in connection with prediction of achievement in the general-mathematics test. Under ordinary circumstances, this result could possibly be accounted for by the fact that there were two ESS tests in the battery. This means that, if both tests are substantial measures of the same factor, a linear restraint would be operating, with the result that the test having the lower correlation with the criterion would tend to have a lower beta weight, in each equation in which they both appear. The correlations between tests for ESS₁ and ESS₂ were .03, .14, .16, and .36, respectively, however, so it appears that possibly only in the algebra groups are the two tests intended for ESS measuring the same factor and they do not correlate sufficiently to cause much concern, except possibly in the accelerated-algebra group. In general, evaluation factors, as a class, showed up rather well in the lists. One current hypothesis is that evaluation means a sensitivity to discrepancies or differences, where fine shadings of information must be discriminated. This would suggest that in teaching algebra some effort might well be focused on sharpening cognitions of mathematical expressions and of their differences.

Some specific findings are also worthy of note in Table 16. For example, it can be pointed out that factor MSI, numerical facility (or memory for symbolic implications, in the broader interpretation of the structure of intellect), shows a systematic decrease in importance from the basic-mathematics group to the accelerated-algebra group. This is as one should expect. The shift very likely means something more than the fact that there is more numerical computation involved in the general-mathematics examination than in the algebra examination. The substantial involvement of MSI for the regular-algebra group may help to support the broader interpretation of the numerical-facility factor as MSI. Although there is some numerical computation involved in the algebra examination, the prominence of the factor may also mean that the student who remembers routine operations with literal expressions well also has an advantage in the examination. In the accelerated group, memory for routine operations may be taken for granted; there should be almost no variance in this respect.

The prominence of factor EMR in two groups (the betas in the basic-mathematics group were zero and in the accelerated-algebra group they were .13 and .11) is worthy of comment. The hypothesis is suggested that, in the evaluation of relations, the basic-mathematics group is rather uniformly low, that the accelerated-algebra group is not quite so uniformly high, but that the two middle groups have more variance with respect to this factor and this shows up in substantial beta weights. The fact that factor EMR is in the semantic category is interesting. It may be that when algebra students evaluate relations they tend to translate them into semantic information, in passing judgment upon them. One might expect the accelerated-algebra group to resort to a more direct approach by applying evaluation of symbolic information, but that group shows betas of only .08 and .10 for factor ESR, the regular-algebra group shows even smaller betas, while the non-college-algebra group has its highest betas for factor ESR. A hypothesis based on a probable decrease in variance in factor ESR as we go up the algebra-course scale hardly seems appropriate.

Of special interest are the divergent-production factors, DSR, DSS, and DSI. At least one of these factors has at least one beta in each of the group lists in Table 16, indicating that divergent-production abilities of one kind or another may play roles at all levels of ninth-grade mathematics. The role of such abilities is probably most conspicuous wherever trial-and-error thinking enters the picture in the student's learning and problem solving. His facility in producing a variety of relations, systems, and implications (deductions) from given information should be of much help. Special exercise in producing alternative solutions to problems should further promote development in this respect.

Reference to divergent-production abilities suggests that we attempt to extract from Table 16 some suggestions as to which categories of abilities in general tend to be weighted most. Not all the categories are equally represented in the total battery of predictors, but this fact can be taken into account. Of the operations categories, a striking deficiency is to be noted for cognition factors. With a possibility for cognition factors to appear as many as 40 times in Table 16, there are only 4 such appearances, or 10 per cent. Only one memory predictor in the battery gives us no basis for concluding concerning the value of memory abilities. The divergent-production, convergent-production, and evaluation classes of factors are represented each in about a third of their opportunities to appear. This little survey, crude as it is, suggests that the expectation that cognition abilities would be relatively important is definitely not supported. It may be, of course, that the right cognitive factors were not represented by tests in this study.

An actual count of the relative frequencies for symbolic and semantic factors shows that they come out about equally effective, contrary to the expectation that symbolic abilities would be more important, at least in algebra. A comparison of factors with respect to kinds of products shows little variation, all being listed in about the same ratios, with relations and implications possibly leading. It should be said that these comparisons cover all four groups of students combined. There might be some differences as between general mathematics and algebra, but the numbers of factors represented and the numbers of their tests are too small to justify carrying the counts to the extent needed to answer this question.

A study like that based upon Table 16 was also made with course grades as criteria, but this step was possible only for the longest equation, including 20 factor variables. Examination of the beta weights shows an even less coherent picture than for the achievement-test criteria. First to be noted is that fewer factors carry the burden of prediction, with the numbers of predictors having betas of .15 and greater appearing at the four mathematics levels as follows: 3, 2, 6, and 4, from basic mathematics to accelerated algebra, respectively. The corresponding frequencies with achievement-test criteria (see Table 16) were: 4, 6, 5, and 5, respectively. The parallel lists of betas, for the two kinds of criteria, do not have a great deal in common. For the two general-mathematics groups, none of the stronger predictors in Table 16 has similar betas in predicting grades. Of the six factors with larger betas for predicting grades in regular algebra, two are the same as in Table 16. These factors are MSI (beta = .33) and NSI (beta = .21). Of the four stronger factors for predicting grades in accelerated algebra, two are also high in Table 16. They are CSR (beta = .20) and CMS (beta = .24).

Thus, there is some confirmation of factors significantly related to achievement in the case of algebra, but not at all in the case of general mathematics. There is considerable room for the appearance of different sets of factors to be involved in the two kinds of criteria, since the correlations of grade criterion with the general-mathematics test score were .64 and .30, in the basic-mathematics and non-college-algebra groups, and of grade criterion with the algebra-examination score, .54 and .51, the traditional scoring of the examinations being used. When we realize that there were a number of teachers involved, each with perhaps different emphases in teaching, examining, and grading, it should not be surprising that there is a somewhat confused general picture. A regression analysis for each teacher separately should be enlightening. Unfortunately, there were too few cases from any one teacher to justify a regression analysis, especially with a relatively large number of predictors. In general, these findings demonstrate the importance of having a uniform criterion in the form of an achievement examination.

Predictions from Standard Tests vs. Factor Tests. — Table 17 summarizes the multiple correlations indicating the goodness of prediction from weighted composites of standard vs. factor tests. The standard tests were combined within the three published batteries as well as over all nine standard tests. The factor tests were combined in two groups of 7 and 13, and a combination of all 20. Multiple R's corrected for bias were used for comparison, since this takes into account the numbers of tests combined in the various equations.

Table 17

Multiple Prediction of Mathematical-Achievement Scores from Weighted Composites of Standard Tests and of Factor Tests*

Prediction Battery	Basic Mathematics	Non-college Algebra	Regular Algebra	Accelerated Algebra
9 standard tests	.60	.53	.22	.74
2 CTMM tests	.34	.40	.18	.37
3 Iowa tests	.53	.31	.20	.62
4 DAT tests	.57	.53	.24	.70
7 factor tests	.42	.56	.27	.51
13 factor scores	.46	.45	.39	.75
20 factor predictors	.48	.54	.38	.74

*The coefficients of multiple correlation given are unbiased, i. e., corrected for shrinkage.

From the first and last rows, it appears that the two kinds of composites, the 9 standard predictors vs. the 20 factor predictors just about break even. The standard battery apparently does better in predicting achievement of basic-mathematics students; the factor battery apparently does better in the regular-algebra group. A more realistic comparison might well be made between each of the three standard composites and the factor composites, however, to fit the case in which a school uses only one aptitude scale. In such comparisons, we find that the DAT combination of four tests does about as well as the factor battery, but the Iowa and CTMM composites appear to do less well. The factor battery of 20 tests, even the battery of 13, possibly requires more testing time than the DAT battery of 4 tests, but if the non-predictive factors were eliminated it might well be found that the factor battery would be more efficient in terms of testing time required.

Increases in Prediction from Combined Standard and Factor Tests. — Although the combined-standard and factor batteries do about equally well in prediction of mathematics achievement in this study, the question may well be asked whether the one prediction duplicates the other. From the fact that there appear to be many new factors represented in the factor tests, it should follow that the equivalent degree of prediction does not mean very much overlapping in prediction, hence a combination of the two batteries should yield even greater accuracy of prediction.

Table 18

Increases in Multiple Correlation from Adding 13 Factor Scores to Each of 3 Standard Batteries of Academic-Aptitude Tests, and F Ratios for Testing Significance of Increases

Predictor Battery	Basic Mathematics		Non-college Algebra		Regular Algebra		Accelerated Algebra	
	R	F	R	F	R	F	R	F
CTMM (2 scores)	.35		.41		.21		.38	
CTMM + 13 scores	.59	1.58	.59	1.60	.54	2.25*	.80	6.06**
Iowa (3 scores)	.55		.34		.24		.63	
Iowa + 13 scores	.65	1.05	.58	1.94*	.54	2.10*	.82	3.23**
DAT (4 scores)	.59		.55		.29		.72	
DAT + 13 scores	.64	0.46	.59	0.48	.55	2.07*	.85	3.36**

* Significant at the .05 level; ** significant at the .01 level.

Table 18 is designed to show how much the 13 factor tests would add to prediction of achievement when combined with each of the standard batteries of tests. Here the biased, or unshrunk, multiple R's are compared, for it is to them that the F test of significance of differences is applied. Examination of the pairs of multiple R's shows uniform gains associated with addition of the factor tests. In general, the higher the level of mathematics course the greater is the difference and the more significant differences there are. There are no significant F's for Basic Mathematics, there is one at the .05 level for Non-college Algebra, three at the .05 level for Regular Algebra, and three at the .01 level for Accelerated Algebra. As forecast from the previous comparisons, the gains are least in the case of the DAT battery and most in the case of the CTMM battery. There is little doubt that highly adequate prediction of achievement in algebra requires going well beyond current academic-aptitude tests.

There is a bit of complication in this particular aspect of the study, stemming from the fact that tests 26 (from the CTMM) and 32 (from the DAT) were included in the composite factor scores for factors CMU and MSI, respectively. This should effect unusually high correlations between the CTMM variable 26 and the score for CMU, on the one hand, and between the DAT score 32 and the factor score for MSI, on the other. The effects of these circumstances upon the multiple-prediction equations are difficult to predict. We have not used these equations in drawing any conclusions concerning the roles of factors. There is a general methodological question whether, in comparing multiple R's based upon standard tests only with R's for combinations of those same tests with factor tests, one should eliminate from the factor battery all factors already represented in the standard tests, provided they are known. To so eliminate them would not permit those factors to have their optimal weights. Not to eliminate them may introduce linear restraints. We leave the methodological question open.

Something can be said regarding the possible contributions that the standard tests make to prediction of achievement in this study, in terms of factors. Examination of the beta weights for the standard tests shows that the CTMM and the Iowa tests contribute mostly in terms of factors CMU (verbal-comprehension) (and possibly NMS and EMR in their CMU tests) and MSI (numerical-facility) variances. The DAT tests also contribute, possibly due to the measurement of factor ESU (evaluation of symbolic units) in its Clerical test and to factor CFR (cognition of figural relations) in its Abstract Reasoning test, a figure-analogies test. Incidentally, it is a bit historically ironic that what, in terms of structure-of-intellect conceptions, is really a test of concrete-reasoning ability has been given the title of Abstract Reasoning! The factors CFR and ESU are not represented in the factor tests used in this study, a fact that undoubtedly accounts for much of the unique contribution of the DAT tests; the fact that they are more valid as a group than the CTMM and the Iowa tests; and the fact that there is significantly increased validity when the DAT and factor tests are combined.

Predictability of Grades.—How predictable are grades as compared with achievement-examination scores? In view of the relatively low correlations between grades and achievement-test scores, we might expect to find some differences in multiple correlations for the two kinds of criteria. Table 19 shows comparisons of multiple r's for the two criteria as predicted from a battery of 9 standard tests vs. 20 factor tests. It is a bit curious that the achievement-test criterion is somewhat more predictable from both standard and factor batteries in the non-college-algebra group but that grades are more predictable than achievement scores in the regular-algebra group, otherwise predictability seems to be about on a par for the two kinds of criteria. Without knowing more about the prevailing grading practices, no ready hypothesis is available. Perhaps more in need of explanation is the lower predictability of the achievement-test criterion for the regular-algebra students, which has been apparent in other comparisons.

Table 19

Multiple Correlations for Predictions of Achievement Scores vs. Predictions of Grades in General Mathematics and Algebra from 9 Standard Predictors vs. 20 Factor Predictors

Criterion	Basic Mathematics		Non-college Algebra		Regular Algebra		Accelerated Algebra	
	S*	F	S	F	S	F	S	F
Achievement scores	.60	.48	.53	.54	.22	.38	.74	.74
Grades	.58	.60	.43	.39	.47	.53	.65	.75

* S stands for standard predictors and F for factor predictors. R's are unbiased.

Predictability of Traditional vs. Construct Scoring.—Table 20 indicates that there is no systematic difference in predictability of traditional vs. construct scoring of the achievement examinations. In view of the extra work involved in construct scoring, there is no advantage from this point of view. It may be that improved procedures in construct scoring can be made to serve useful purposes other than that of increased predictability of total scores.

Table 20

Multiple Correlations for Prediction of Construct Scores vs. Traditional Scores of Achievement Tests from 9 Standard Predictors vs. 20 Factor Predictors

Type of Score	Basic Mathematics		Non-college Algebra		Regular Algebra		Accelerated Algebra	
	S*	F	S	F	S	F	S	F
Traditional Construct	.60	.48	.53	.54	.22	.38	.74	.74
	.62	.60	.45	.60	.24	.31	.67	.80

* S stands for standard predictors and F for factor predictors. R's are unbiased.

DISCUSSION

In this report, there remain only a few general reflections and appraisals to be offered, concerning the general approach to the problem and the way things worked out. From this overview, can anything be gained concerning future attacks on the problem of understanding the intellectual resources needed for mastering ninth-grade mathematics and of predicting achievement in that area of instruction?

Advantages of a Theoretical Approach

Predictive-validation studies could, of course, proceed without concern for construct validity. This is the highly empirical manner in which most predictive-validation studies have proceeded. Some investigators prefer to stay close to empirical data and they may even be reluctant to conceive of underlying variables, such as factors. The writers of this report are committed to an approach that is also rational; that attempts to take advantage of things that a general theory has to offer.

There are several advantages of working from the basis of a general theory, besides that provided by the general frame of reference that it furnishes. Most obvious is the very broad scope of potential abilities that are suggested by the model. Even the study by Majewska (1960), which introduced many more different kinds of tests than the other investigators, did not have the scope that is provided by the structure-of-intellect model. Furthermore, the SI model is a systematic organization of the abilities, which makes possible a representative sampling of abilities to be brought into a validation study. Owing to limitations of experimental time, there must be some sampling of abilities to include, even under the best circumstances.

The meaningfulness of the factors in the SI model is another significant advantage, which carries beyond validation studies. If we establish the fact that each of several SI abilities accounts for variances in good criteria of mathematical achievement, we also have information that should be useful in teaching and examining practices. As a general, educational outcome, we should know which of the intellectual abilities are exercised in the study and mastery of a subject like algebra. Questions of formal discipline should thus be meaningfully answered. In connection with the diagnosis of student difficulties, the assessment of students in the various pertinent factors should be of considerable help to the counselor and the teacher. These suggestions are a few that might be offered on how to use information concerning the relevant intellectual abilities involved in learning in algebra.

Relevant and Irrelevant Factors

What have we learned from this investigation that should guide future studies of this kind? The answers to this question should point out the most relevant factors that should surely be included in future studies, the ones that might well be dropped from consideration, and others that might well be added for study.

For the prediction of achievement in general mathematics, including Basic Mathematics and Non-college Algebra, the factors of MSI, NMS, EMR, and ESR are strongly indicated. Taking both the Regular-Algebra and Accelerated-Algebra courses together, the relevant factors most strongly indicated are NSS, DSR, and EMR, from the three categories of divergent production, convergent production, and evaluation. As suggested before, the seeming potential for divergent-production abilities must be due to the need for multiple-response, trial-and-error behavior that is helpful in arriving at the right answer. The convergent-production abilities must be helpful in circumventing the need for divergent production, where it is possible to produce the correct solution immediately. There should be no doubt of the relevance for relations and systems in solving algebraic problems. In the two lists just given, it is noteworthy that, except for factor EMR, the leading factors are non-overlapping for general-mathematics and algebra achievement. It is also noteworthy that except for factors MSI, NMS, and EMR, the factors in these two lists are probably not covered in standard tests.

In relation to the findings from the investigators mentioned early in this report, which of their findings have been supported by this study and what additional knowledge has been gained? Considering all kinds of mathematics courses and criteria together, the factors found to have some promise before and also in this study are: CSR, CMS, MSI, NSR, NSS, NMS, and EMR. We found an instance of a negative beta for a CMU score and no apparent

relevance at all for a CSS score. Factors not known to be validated before but showing at least some signs of promise are: DSR, DSS, DSI, NSI, ESR, ESS, EST, and ESI. Two factors for which some relevance was expected but none was found, were CSI and NST.

It is not so easy to say which of all the factors involved in this study are actually irrelevant, for in replications of this investigation one does not know but that certain factors that seem irrelevant in this case will show relevance elsewhere. There were a few surprises. Factors CSS and CSI, which were expected to be relevant, do not appear at all in Table 16. This finding is in line with the general paucity of cognition abilities in the lists, and it contributes to that general observation. We should not recommend that they be dropped from further consideration, however, because they still have logical appeal. It should be recalled that earlier investigators had found some relevance for CSS. CSI has had only one opportunity, in the present study.

Transformation abilities were not at all well represented in this study, and the two that were included did not appear very promising. Again, logic suggests that they be given further consideration and that other transformation abilities be brought into such studies as soon as tests of them become available.

A couple of apparently relevant factors not included among the factor tests seem to have some claim to future attention. They are suggested in the results from the DAT tests. They are the factors CFR, probably prominent in the test called Abstract Reasoning, and ESU, probably prominent in the Clerical test. It is noteworthy that these two instances go outside the symbolic category, in the first case, and outside the four product categories to which we restricted ourselves very much, in the second case. Thus, figural abilities, and abilities involving units, are not beyond the realm of possibility in future studies. There was no result that had any known bearing upon the relevance of abilities for dealing with classes, but, in courses that involve modern algebra, factors involving classes should certainly be brought into consideration.

From all this, it is clear that there is much to be done before we exhaust the possibilities of aptitude factors related to achievement in ninth-grade mathematics. Our results indicate the great complexity of the problem. Instead of narrowing the search for valid components of predictive batteries, the results have only broadened the search. The possibility of multiple correlations as high as .80 (unbiased) with only a few predictive factors suggests that, with more relevant factors added, a battery that would yield highly useful predictions should be quite possible.

SUMMARY AND CONCLUSIONS

The major purpose of this investigation was to determine to what extent achievement in ninth-grade-algebra and general-mathematics courses can be accounted for in terms of certain factors represented in the structure-of-intellect model. There was secondary interest in knowing how predictive validity for measures of those factors compares with that available from standard academic-achievement tests and whether prediction could be improved by combining factor scores with standard-test scores. The emphasis was upon construct validity of predictors and criteria as predictive validity was being investigated. The aptitude factors emphasized were mostly in the area of symbolic information, for the operations of divergent and convergent production and evaluation, and for products of relations, systems, transformations, and implications. Not all factors that might well have been investigated were included, many of them still being unknown.

Approximately 600 ninth-grade students in one of the Los Angeles County high schools served as subjects, for approximately 400 of whom test and criterion data were complete. The students had been administered the California Test of Mental Maturity, the Iowa Test of Basic Skills, and the Differential Aptitude Tests, from which some part scores were used.

Four mathematics courses were involved, including Basic Mathematics, Non-college Algebra, Regular Algebra, and Accelerated Algebra, the first two courses coming under the heading of General Mathematics. About equal numbers of subjects were available in each course; but not sufficient numbers with which to perform a cross-validation study.

Of the factor tests administered, 25 were factor analyzed (along with three marker tests from the standard batteries) in the general-mathematics and algebra populations separately, in order to determine whether the tests were measuring the same factors as in previous adult populations and to provide a basis for combining tests to obtain factor scores. For the 13 factors, which were identified as being the same in the two analyses, combinations of tests were effected to provide 13 factor scores. An extra set of seven tests, representing six hypothesized factors, four of which have not yet been demonstrated, mostly in the evaluation category, were also utilized.

Two achievement examinations, in general mathematics and algebra, were administered at the end of the year's work in mathematics. Multiple-regression equations were derived, predicting both the achievement-examination and course-grade criteria from combinations of standard tests, factor tests, and combinations of both kinds of predictors. The roles of separate factors in these predictions were inferred from the beta weights of the equations.

The results led to the following general conclusions:

1. The higher the level of the mathematics course, the larger the number of factors that appear to be involved, from six in Basic Mathematics to ten in Accelerated Algebra.
2. The pattern of more predictive factors varies according to the course level, but certain factors are significantly related in different courses and certain systematic differences occur as between general mathematics and algebra in their apparent demands on the factors.

3. On the whole, composites of standard academic-aptitude tests and composites of factor scores do about equally well in prediction, but for somewhat different factorial reasons, with some tendency for standard tests to do better in predicting in general mathematics and factor tests to do better in predicting in algebra.

4. When factor tests are added to each of the three standard-test combinations, multiple prediction increases significantly for algebra courses but not for general-mathematics courses.

5. Grades and achievement-test scores are about equally predictable from either standard or factor tests, but the factor picture of course-grade criteria is usually inconsistent with that for achievement-test criteria, especially for general mathematics.

6. Construct scoring, with detailed point systems along factorial lines, did not provide a total criterion that was any more predictable than the traditional scoring of one point for each problem correctly solved.

7. The problem of aptitude for success in ninth-grade mathematics is even more complex than was anticipated, and will require a broader sampling of the intellectual abilities before it is solved.

8. A test battery that will yield a high level of prediction of mathematics achievement appears to be possible, when enough of the relevant factors are included.

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APPENDIX A

DESCRIPTION OF TESTS

(The tests are not listed in the order in which they were administered. The tests developed for the factor analysis are listed alphabetically as variables 1 through 25. Variables 26 through 34 are parts of standard, published tests. Scores from these sources were obtained from school records. Variables 36 and 37 are the specially-constructed criterion measures for general mathematics and algebra. Variables 38 through 45 are tests originally developed for factor-analytic study in another project. They were included in the present battery for prediction purposes only.)

1. **Alternate Additions** - DSR01B. Show how numbers in a set may be related in obtaining the same total with any or all given numbers, when only addition is permitted and only the given numbers may be used to obtain the same total in different ways.

Sample Item: Given: 1, 2, 3, 4 Obtain: 7 as a sum.

Possible responses: $3 + 4 = 7$

Score: one point for each different solution within items. Parts: 2; items per part: 8; working time: 6 minutes.

$1 + 2 + 4 = 7$

$3 + 3 + 1 = 7$ (not acceptable)

2. **Best Trend Name** - EMR01A. Choose the word that best describes the order of four given words.

Sample Item: Given words: horse - push cart - bicycle - car.

Given trend names: A. speed
B. time
C. size

Answer: B

Score: one point for each correct item. Parts: 2; items per part: 10; working time: 6 minutes.

3. **Camouflaged Words** - NST01A. Find within a meaningful sentence a group of consecutive letters that, in the given order, spell the name of a sport or game.

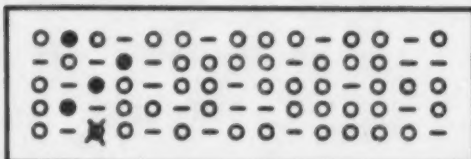
Sample Item: COWARDICE IS NOT A SOLDIERLY ATTRIBUTE.

Answer: DICE

Score: one point for each correct response, one camouflaged word per item. Parts: 2; items per part: 10; working time: 8 minutes.

4. **Circle Reasoning**¹ - CSS01C. Discover the principle by which one circle is blackened in each of four rows of mixed circles and dashes.

Sample Item:



Answer: The second circle from the left is blackened, so the appropriate circle is marked in the last row.

Score: one point for each item. Parts: 1; items per part: 14; working time: 8 minutes.

5. **Correlate Completion II** - NSR01A. Supply a word that bears the same relation to the single word as the relation between the words in the two given pairs. The relation between the words in each pair is based on the letters of the words and not on their meanings.

Sample Item: am - ma not - ton tool - ?

Answer: loot

Score: one point for each correct word written. Parts: 2; items per part: 20; working time: 12 minutes.

6. **Form Reasoning**¹ - NSI02B. From the table, find a form that is equal to the three given forms.

Table:

☆ ○ = □	○ ☆ = □	○ ◇ = +	+ □ = △
☆ + = ○	○ □ = ☾	◇ ○ = +	+ ☆ = ○
□ + = △	○ + = ☆	☾ + = ◇	+ ○ = ☆
□ ○ = ☾	○ ☾ = 8	☾ ○ = 8	+ ☾ = ◇

Sample Item: ☆ ○ + = ☆ □ ☾ △ +

Answer: According to the table presented above, the first two figures, the combination of star and circle, is equal to a square. If a square is put in place of the star and circle, a square and a cross remain, which are equal to a triangle. Therefore the correct answer is a triangle.

Score: one point for each correct response. Parts: 2; items per part: 10; working time: 4 minutes.

7. **Letter-Number** - NSI03A. Find the relationship between letters and numbers and use the relationship to find the number that corresponds to new letters.

Sample Item: no = 56
po = 76
mo = ?

Answer: mo = 46

Score: one point for each correct response. Parts: 2; items per part: 8; working time: 8 minutes.

8. **Letter Series** - NSR02B. Find the rule of order in a series of letters and then fill in the blanks with the letters that would fit the rule.

Sample Item: E F E F E F E _ _

Answer: F E

Score: one point for each correct response. Parts: 2; items per part: 15; working time: 8 minutes.

9. **Letter Triangle** - CSS02A. With letters arranged systematically within a triangular pattern, which letter should appear in a marked place?

Sample Item:

	a	
b	c	
d	e	f
?		

A h
B j
C g
D f
E a

Answer: C

Score: one point for each correct response. Parts: 2; items per part: 8; working time: 12 minutes.

¹ Reproduced by permission from a test by Robert Blakey.

10. Matched Verbal Relations - EMR02A. Choose one of four pairs of words that has the same relationship between words as that of the given word pair.

Sample Item: BIRD - SONG

Answer: 2

Score: one point for each correct response. Parts: 3; items per part: 10; working time: 9 minutes.

1. fish - water
2. man - letter
3. pianist - piano
4. horse - ranch

11. Necessary Facts - CMS04A. Determine what information is needed to attain solutions for given arithmetic problems in which needed facts are missing.

Sample Item: A rectangular tank is being built to hold water. It is to be 5 feet high and 9 feet long. How many cubic feet of water will it hold?

Answer: The width of the tank (needed fact).

Score: one point for each correct response. Parts: 2; items per part: 8; working time: 8 minutes.

12. Number Rules - DSR02B. Starting with one given number, arrive at a second given number, by means of single arithmetic operations.

Sample Item: Starting with 2, get 6 in a variety of ways. Possible answers: $2 + 4 = 6$, $2 \times 3 = 6$, $2 + 2 + 2 = 6$

Score: one point for each solution that uses different rules. Parts: 2; items per part: 5; working time: 10 minutes.

13. Numerical Operations, Guilford-Zimmerman Aptitude Survey, Part III² - MSI01A. Add, subtract, or multiply numbers.

Score: number right minus number wrong. Parts: 1; items per part: 132; working time: 8 minutes.

14. Picture Arrangement³ - NMS02A. Given four pictures from a comic strip in scrambled order, put them in the correct temporal order.

Score: one point for each correct order with no credit for partially correct order. Parts: 4; items per part: 4; working time: 6 minutes.

15. Right Order Test - NSS01B. Starting with one given number, do three given numerical operations in the right order to obtain a specified result.

Sample Item: Start with 6 a) +3 Answer: the right order is b, a, c.
and obtain 18 b) $\div 2$
 c) $\times 3$

Score: one point for each correct order minus one-fourth the number of incorrect orders. No credit is given for partially correct orders. Parts: 2; items per part: 10; working time: 8 minutes.

16. Seeing Trends II - CSR01A. Describe a trend based upon relations of letters in a group of words.

Sample Item: rated crate morning dearth separate

Answer: The letter "r" moves one place to the right with each word.

Score: one point for each correct response. Parts: 2; items per part: 12; working time: 16 minutes.

17. Sentence Order⁴ - NMS03A. Arrange three sentences in a sensible order.

Sample Item: 2 She bought some food at the market
 3 She returned home and cooked some of the food she had bought
 1 She went to the market

Score: one point for each correct ordering; no credit for partially correct orderings. Parts: 2; items per part: 10; working time: 6 minutes.

18. Ship Destination Test, Form A-2⁵ - CMS. State how many miles a ship travels from one point to another considering distance between points along the way, directions, strength of wind and current, and starting position.

Score: one point for each correct response. Parts: 1; items per part: 48; working time: 15 minutes.

19. Sign Changes - NSI01A. Solve simple arithmetic problems in which the operation sign is changed according to a set of rules.

Sample Item: Rules: instead of +, you -
 instead of -, you $\times$

Problems: $6 + 2 = \underline{4}$
 $3 - 6 = \underline{18}$
 $4 + 3 = \underline{1}$

Score: one point for each correct solution. Parts: 3; items per part: 8, 16, and 16; working time: 2 1/2 minutes.

²Adapted with permission from a test of the same name copyrighted by the Sheridan Supply Co., Beverly Hills, Calif.

³Adapted from Dorothy C. Adkins' adaptation of the comic strip "Louie," Copyright 1948 and 1949, Press Features, Inc. Used by permission.

⁴Adapted from a test of the same name developed at the University of North Carolina under contract with the Adjutant General's Office.

⁵Adapted with permission from a test of the same name copyrighted by the Sheridan Supply Co., Beverly Hills, Calif.

20. Symbol Grouping - CSI01A. Rearrange scrambled symbols in a specified systematic order as efficiently as possible.

Sample Item: Given: X - OX - X By moving one or more adjacent symbols at a time, arrive at the order XXX--O

Answer: (The minimum number of moves is two.)

Score: two points for the most efficient rearrangement possible, one point for a rearrangement less efficient, and no points for an inefficient or incorrect rearrangement. Parts: 2; items per part: 8; working time: 10 minutes.

21. Word Changes - NSS02B. Given a set of words, each containing the same number of letters, one designated as first and another designated as last, arrange the remaining words so that exactly one letter is changed from one word to the next.

Sample Item: SET

3

4

1

2

CRY

1. DAY
2. DRY
3. SAT
4. SAY

Score: one point for each correct order; no credit for partially correct orders. Parts: 2; items per part: 6; working time: 8 minutes.

22. Word Linkage - EMR03A. Choose from a list of three words, the one which is related to the two given words by virtue of two different meanings.

Sample Item: Jewelry - Bell

Answer: 3

Score: one point for each correct response. Parts: 2; items per part: 15; working time: 8 minutes.

1. ornament
2. jingle
3. ring

23. Word Patterns - CSI03B. Arrange a list of short words efficiently in a kind of crossword-puzzle design.

Score: credit is given for efficiency of space usage by the given short words. Parts: 2; items per part: 3; working time: 12 minutes.

24. Word Relations - CSR02B. Recognize the same relation between words in each of two pairs, then complete a third pair using the same relation, alternative answers being given.

Sample Item:

on no
top pot
part ?

1. art
2. pat
3. rapt
4. tar
5. trap

Answer: 5

Score: one point for each correct response. Parts: 3; items per part: 10; working time: 12 minutes.

25. Word Transformation - NST02A. Regroup the letters of words in a phrase so as to make another phrase.

Sample Item: (with answer marked) THE RED OLIVE

Score: one point for each correct regrouping of letters to form a new word. Parts: 1; items per part: 20; working time: 6 minutes.

26. California Test of Mental Maturity - Language MA. (CMU)⁶

27. California Test of Mental Maturity - Non-Language MA. (MSI and CMU)

28. Iowa Reading Comprehension - Test R. (CMU)

29. Iowa Arithmetic Concepts - Test A-1. (CMU and MSI)

30. Iowa Arithmetic Problem Solving - Test A-2. (MSI and CMS)

31. Differential Aptitude Test - Verbal Reasoning. (CMU, CMR, and NMR)

32. Differential Aptitude Test - Numerical Ability. (MSI)

33. Differential Aptitude Test - Abstract Reasoning. (CFR)

34. Differential Aptitude Test - Clerical Speed and Accuracy. (ESU)

35. Sex. Boys were given a "score" of 1; girls were given a zero.

36. Mathematics Achievement Test - B1. Test designed to measure achievement in 9th-grade mathematics based on the stated course objectives and standard mathematics achievement.

Score: The traditional scoring gave one point for each correct response to a problem. A four-choice multiple-choice section was scored one point for each correct response minus one-third the total number of incorrect responses. Construct scoring of this test gave one point for each correct operation in the solution of each problem. Parts: 1; items per part: 30; working time: 40 minutes.

37. Algebra Achievement Test. Test designed to measure achievement in beginning algebra based on the stated course objectives and standard algebra achievement.

Score: The traditional scoring gave one point for each correct response to a problem. A four-choice multiple-choice section was scored one point for each correct response minus one-third the total number of incorrect responses. Construct scoring of this test gave one point for each correct operation in the solution of each problem. Parts: 1; items per part: 30; working time: 40 minutes.

⁶Structure-of-intellect factor contents hypothesized as being measured by published tests are enclosed in parentheses.

38. Abbreviations - ESI01A. Choose the word that the given abbreviated word most likely implies.

Sample Item: crnt A. crescent Answer: C Score: one point for each correct response. Parts: 2; items per part: 15; working time: 8 minutes.
 B. coronation
 C. current

39. Condensations - EST01A. Choose the better of two shorthand alternatives, choice to be based on the unique meaning of the shorthand.

Sample Item: WALK THROUGH GREEN FORESTS. A. Wlk thrgh grn frts. Answer: B (Alternative B is less ambiguous.)
 B. Wlk thrh gren frsts.

Score: one point for each correct choice. Parts: 2; items per part: 10; working time: 6 minutes.

40. Letter-Number Scales - ESS01A. Given the numerical values of two letters of the alphabet, estimate the numerical value of a third letter.

Sample Item: b = 7 A. 34 Answer: B (No answer is correct, but alternative B is closest to correct.)
 g = 9 B. 17
 y = ? C. 20 Score: one point for each keyed answer. Parts: 2; items per part: 15; working time: 6 minutes.

41. Limited Words - DSI02B. Given two words, make up additional pairs of words (anagrams) using all the letters in the given pair.

Sample Item: Given: SHIRT BEAN Possible answers: hairs bent
 bears thin

Score: one point for each acceptable pair produced. Proper names and slang are not acceptable. Parts: 2; items per part: 4; working time: 10 minutes.

42. Most Similar Sets - ESS02A. Choose which of two sets of symbols is most like a given set.

Sample Item: Given: A2 A3 A4 A. B B B Answer: B
 B. a5 b6 c7

Score: one point for each keyed response. Parts: 2; items per part: 10; working time: 8 minutes.

43. Number Combinations - DSS02B. Write several different equations using only given numbers and given operations.

Sample Item: Given numbers: 2, 3, 4, 5, 6
 Given operations: + and x
 Possible equations: $5 = 2 + 3$ Score: one point for each correct equation. Parts: 2; items per part: 3; working time: 8 minutes.
 $3 + 4 = 2 + 5$
 $2 \times 4 = 3 + 5$

44. Sign Changes II - ESR01B. Make the sign changes necessary to make an expression into an equation.

Sample Item: Expression Sign Changes Answer: C
 $3 + 1 = 6 \times 2$
 A. Instead of + you -
 B. Instead of + you x
 C. Instead of x you -
 D. Both A and C

Score: one point for each correct answer. Parts: 2; items per part: 10; working time: 8 minutes.

DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE
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Project Description

Structure of Intellect Abilities in Algebraic Thinking

Problem.--This research investigates the structure of intellect abilities necessary to algebraic thinking.

Objectives.--The objectives are (1) to identify the intellectual factors necessary for successful algebraic thinking; (2) to discover whether the factor structure for algebraic ability differs from that of 9th-grade general mathematics; (3) to discover whether the appropriate structure-of-intellect tests will serve as predictors of success in 9th-grade algebra.

Procedures.--Structure of the intellect tests will be administered to 500 9th-grade students in algebra and general mathematics courses. Criterion tests covering the usual content of these courses will be given at the end of fall semester and spring semester. Factor analyses will be made to compare the factor structure of boys with that of girls, and the structure of algebra students with that of general mathematics students. Composite factor scores will be derived and used in a multiple-regression procedure to predict course grades and scores on criterion tests. Profiles of students especially able in algebra will be investigated.

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Project No. 1342.
Duration: two years.
August 1961 to August 1963.